Application of Graph Theory to image segmentation

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CHAPTER 1- INTRODUCTION

1.1 Background of the study

Graph theory is becoming a significant tool applied widely in the numerous research areas of mathematics, science, research and technology. This graph theory uses graphs were vertices or points are collected and connected to the edges which are also known as lines. In graph theory, some are directed graph where lines have a single arrow which denotes in specific direction alone. In some other graph, edges are not located which is it does not have any edges or sometimes lines may overlap but is have the same vertex alone. The combination of graph theory is previously applied in the area of mathematics alone later it has been applied to a wider range of applications. Graph theory has been applied in the research areas of biochemistry, computer science, and operations research, electrical engineering field with a specific application of genomics, computation, and algorithm, scheduling, network communication respectively. Some of the recent research areas for application of graph theory is DNA sequencing security in a computer network through the use of minimized vertex graph. Some other applications of graph theory are minimizing traffic congestion, creating road network, evaluating the impact of environmental data and so on.

1.1.1 Application of Graph Theory on Image Processing

Peng *et al*. (2011) Image segmentation is a fundamental yet still challenging problem in computer vision and image processing (Peng *et al*., 2013). In particular, it is an essential process for many applications such as object recognition(Yu *et al*., 2002; Russell *et al*., 2006), target / object tracking(Kap-Ho Seo & Ju-Jang Lee, 2005; Lam & Yuen, 1998), content-based image retrieval and medical image processing(Funka-Lea *et al*., 2006; Grady *et al*., 2006; Jolly *et al*., 2009), image reconstruction (Favaro & Soatto, 2004; Bleyer & Gelautz, 2005) and so on. Generally, the image segmentation is to partition an image into a certain number of pieces which have coherent features (color, texture, etc.) and in the meanwhile to group the meaningful pieces together for the convenience of perceiving (Funka-Lea *et al*., 2006). In many practical applications, as a large number of images are needed to be handled, human interactions involved in the segmentation process should be as less as possible. This makes automatic image segmentation techniques more appealing. Moreover, the success of many high-level segmentation techniques (e.g. class-based object segmentation (Grady *et al*., 2006; Jolly *et al*., 2009) also demands sophisticated automatic segmentation techniques.In spite of several years of research, the generalized solution to image segmentation problem is still a very tough task because segmentation is inherently ill-posed. Among different segmentation methods, graph theoretical approaches have many good features in practical applications (Chang *et al*., 2015)(Dezso *et al*., 2012). It organizes the image elements into mathematically well-defined structures, making the formulation of image segmentation problem more desirable and the computation more efficient. Graph partitioning methods can effectively be used for image segmentation. In these methods, the image is modeled as a weighted, undirected graph. Usually, a pixel or a group of pixels are associated with nodes and edge weights define the (dis)similarity between the neighborhood pixels. The graph (image) is then partitioned according to a criterion designed to model "good" clusters. Each partition of the nodes (pixels) output from these algorithms are considered an object segment in the image.

The term graph theory is a tool towards describing the image processing algorithms(Dezső *et al*., 2012). Its explosive growth in recent years due to its role as an essential structure underpinning modern applied mathematics – computer science (Shirinivas *et al*., 2010a), combinatorial optimization, and operations research in particular – its increasing application in the applied sciences (Bondy & Murty, 2008), data mining, image segmentation, clustering, networking, the image capturing, etc. Also used in network modeling,graph coloring concept in resource allocation, scheduling, database design concepts, traveling salesman problem and resource networking. This results in the advancement of new theories and algorithms that can be used in tremendous applications.One of the major application of graph theory is its application in image segmentation. Peng *et al*. (2013) Additionally, the segmentation problem is then solved in a spatially discrete space by the efficient tools from graph theory. One of the advantages of formulating the segmentation on a graph is that it might require no discretization by virtue of purely combinatorial operators and thus incur no discretization errors. For instance, the versatility of graphs makes them an indispensable tool for the design and analysis of communication networks, in mathematically, the graph theory is the study of the graph that is used to model pairwise relations between the objects from a certain collection. A term graph is a collection of vertices/nodes and collection of edges that connect pairs of vertices. It may be undirected (the distinction between the two vertices associated with each edge) or directed from one vertex to another (Grama *et al.*, 2012).

1.2 Problem statement

Over the past few decades, the image segmentation and grouping are a challenging task for computer vision (Felzenszwalb & Huttenlocher, 2004). Among different segmentation methods, graph theoretical approaches have many useful features in practical applications. It organizes the image elements into mathematically well-defined structures, making the formulation of image segmentation problem more desirable and the computation more efficient. The graph-based image segmentation is a highly effective and cost-effective way to perform image segmentation.But, it not been implemented in an efficient manner. Additionally, the previous research fails to provide versatile graph-based algorithms for a broad range of practical applications (Peng *et al*. 2013).

Furthermore, the numerous applications and a huge amount of medical image data need sophisticated software that combines high-level graphical user interfaces as well as robust and fast interactive image analysis tools. For this purpose, the graph-based image segmentation method is used. The main advantage of using this approach for formulating the segmentation on a graph and does not require discretization by the features of Combinational operators. Additionally, discusses the graph theoretical methods for image segmentation, where the problem is modeled in terms of partitioning a graph into several sub-graphs such that each of them represents a meaningful object of interest in the image. Furthermore, compare the efficiency and effectiveness of various graph-based image segmentation algorithms in medical image classification. These methods have generated significant interest in the imaging community. The study focuses on the following properties: minimal spanning trees, normalized cuts, Euler graph, and Iterated Graph cut.

1.3 Research aim

This study mainly aims to develop the framework for image segmentation by applying graph theory. The focus of segmentation is to simplify or modify the representation of an image into somewhat that is more significant and easier to analyze. Image segmentation is normally used to trace objects and boundaries (lines, dots, curves, etc.) that can occur in images.

1.4 Research objective

The main focus of this research is to applications of Graph Theory in Image Segmentation. In specific, image segmentation is to simplify the representation of a picture into something that is more genuine and simpler to understand. It is essentially used to discover the location of objects, boundaries, lines and so on in the digital images. More precisely is the process of assigning a label to every pixel in the image such that pixels with the same label share certain visual characteristics or features.

The primary objective of the this research is,

- To review of various image segmentation techniques on graph theory method which are flexible and computationally more efficient
- To develop and simplify/modify the representation of an image into more significant and easier way in order to analyze the image segmentation by applying graph theory.
- To segment the image using two different kinds of local neighborhoods in constructing the graph, and illustrate the results with both real and synthetic images.
- To analyze the quality of the image by using various segmented results. This used to analyze the efficiency and accuracy of the segmentation process.

1.5 Significance of the study

This study covers the importance of graph theory and its application towards image segmentation. The focus of this section is on segmentation methods that are based on detecting sharp, local changes in intensity. The three types of image features in which are interested as points, edges, and lines. Subsequently, graph theory is a very efficient tool in image processing which is used for filtering, segmentation, clustering and classification purposes. Thus, this theory is becoming a perfect representation of image processing and analysis.

1.6 Chapterization

The entire research is divided into Six chapters.These are, Chapter I is Introduction, described the background of the study, which includes a brief description of graph theory and its application in image segmentation. Furthermore discussed the research problem, objective, and significance of the study.Then Chapter II is Literature Review, which begins with the concept of graph theory and further explains an overview of segmentation and application of graph theory. Moreover discussed the previous studies related to various methods (Image cluster, analysis, segmentation, wavelet transform and feature extraction) and application (remote sensing, medical application, computer vision) .Chapter III is Research Methodology, which recognizes the research questions. It explains procedures and the objectives to carry out the research. A detailed description of all the research methods including sampling procedure, data collection and type of analysis to be used for the data is elaborated in this section. Chapter IV is discussed the experimental methods. Further, analyzed the performance of proposed approach using image quality analysis approach. Chapter V discussed the simulation results using Matlab software with various images like Computed Tomography (CT), Magnetic Resonance (MR), Ultrasounds (US) images. Finally, Chapter VI discussed findings and conclusion of this research.

CHAPTER II: LITERATURE REVIEW

2.1 Introduction

In real world scenario, it can be illustrated diagrammatically with a set of points joined with lines Sasireka and Kishore (2014). Normally, the points could represent computer terminals in a computer network with lines representing communication links. A mathematical abstraction of situations which focuses on the way in which the points were connected give rise to the concept called graph (Bondy & Murty, 1976; Harary, 1969). Graph theory is one of the recent research areas of modern mathematics which has witnessed a magnificent growth due to some applications in computer and communication, molecular physics and chemistry, social networks, biological sciences, computational linguistics, and in other numerous areas. In graph theory, one of the extensively researched branches is domination in the graph(Haynes *et al.*, 1998). In recent years, graph theory exhibits a spectacular growth has been witnessed due to its wide range of applications in classical algebraic problems, optimization problems, combinatorial problems, computational problems, etc. This is mainly due to the rise of some new parameters that has been developed from the basic definition of domination.

2.2 History of graph theory

Among various fields of research, mathematics plays a vital role where graph theory is widely used in structural models (OK & SD, 2015). This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. Graph theory exhibits significant growth in the recent mathematics for solving a complex problem which is stated by Koinsberg Bridge in 1735 (Shirinivas *et al.*, 2010b). This problem leads to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of a complete graph and bipartite graph and Kuratowski proved that they are planar using recreational problems. The concept of a tree, (a connected graph without cycles (Deo, 19990) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. In the year of 1856, Thomas. P. Kirkmanand William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. Even though the four color problem was invented, it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Graph theory is a branch of discrete mathematics where mathematics and computer science, graph theory is the study of graphs which are mathematical structures used to model pairwise relations between objects Singh and Vandana (2014). For solving a wide range of problems, graphs are widely used because it gives an intuitive manner before presenting formal definition. To analyze the graph theory application two problem areas are considered.

1. Classical problem
2. Problems from applications

In graph theory, the basic classical problem is defined with the help of the graph theory as connectivity, cuts, paths and flows, coloring problems and theoretical aspect of graph drawing Singh and Vandana (2014). Whereas these problems from application particularly emphasis on experimental research and the implementation of the graph theory algorithms. Graph drawing (Nishizeki *et al*., 2000) is a key topic in implementation point of view because the automatic generation of drawing graph has important applications in key computer science technologies such as database design, software engineering, circuit designing, network designing and visual interfaces.

2.3 Definition of Graph theory

Graph theory is used in the field of mathematics and computer science using graphs in the form of mathematical structures pairwise relations models between objects from a certain collection. This graph is defined as a set of objects called vertices, points or nodes connected by links called lines or edges. Through the graphical model nodes and the second set of items called edges where a graph is defined as a relationship between such sets. In this set, each edge joins a pair of nodes where graphs are represented graphically by drawing a dot for every vertex, and drawing an arc between two vertices if they are connected by an edge through the use directed arrow drawing arrows. Every edge can give a real value which means that a graph is extended with a weight function. In the case, when a graph presents set of nodes, the weight function is a length of every mode which is stated as a weighted graph.

A graph G is a mathematical structure used to model pairwise relations between objects from a certain collection. A graph in this context refers to a nonempty set of vertices and a collection of edges that connect pairs of vertices. The set of vertices is usually denoted by V (G) and the set of edges by E (G). The edges can be directed or undirected; it depends on the example. A graph with all directed edges is called directed graph. Otherwise it is called undirected. In a proper graph, which is by default undirected, a line from point u to point v is considered to be the same thing as a line from point v to point u. In a digraph, short for a directed graph, the two directions are counted as being distinct arcs or directed edges. If a graph G is undirected, then there is no distinction between the two vertices associated with each edge. In a directed graph its edges may be directed from one vertex to another.

2.4 Application of graph theory

Patel and Patel (2013) Graphs are used to model many problems of the real word in the various fields. Graphs are extremely powerfull and yet flexible tool to model. Graph theory includes many methodologies by which this modeled problem can be solved. Authors of the paper have identified such problems, some of which are mentioned in this paper.

Shirinivas *et al*. (2010) Graph theoretical concepts are widely used to study and model various applications, in different areas. They include study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist, and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affects other species. Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. It is also used in modeling transport networks, activity networks, and theory of games. The network activity is used to solve a large number of combinatorial problems.

2.4.1 Application in computer science

2.4.1.1 Database designing

In database designing graphs are used as graph databases (Bordoloi & Kalita, 2013). A graph database uses graph representation with nodes, edges, and properties to represent and store data. This graph structure has a key role in designing database because it gives fast implementation process using different functionality and properties of the graph structure .A graph database uses as:

- Storage system that provides index-free adjacency
- Analysing tool for interconnection
- Powerful tool for graph like-query

Graph databases are often faster for associative data sets that map more directly to the structure of object-oriented applications.

2.4.1.2 Software engineering

The graph has many applications in software engineering. For example: during Requirements Specification, Data Flow diagrams are used where vertices represent transformations and edges represent the data flows. During Design phase, graphical design is used for describing relations among modules; while during Testing, the control flow of a program associated with McCabe's complexity measure which employs directed graphs for addressing the sequence of executed instructions and etc. Even Software Process Management has also applications of network diagrams which involves graph algorithms.

2.4.1.3 Network system

Graph theory has wide application in the field of networking. To analyze the graph theory application in networking two areas are considered: graph based representation and network theory. Graph based representation has many advantages such as it gives a different point of view; it makes the problem much easier and provide a more accurate definition. Whereas network theory provides a set of techniques for analyzing a graph and applying network theory using a graph representation. The term graph and network are equal. Both refer to a type of structure in which there exists vertices (i.e. nodes, dots) and edges (i.e. links, lines). There are numerous types of graphs and networks which yield more or less structure. These two terms can be differentiating on the basis of their utility. The term graph is used in mathematics whereas the term network is used in physics.

2.4.1.4 Computer hardware

In computer hardware, graph theory concepts are used to model the limitation of the physical layer. Graph theory concepts are used in hardware world to provide:

- Register allocation by graph coloring
- Representation of instruction sequences by graphs by adjacency matrix
- In instruction parallel processing
- The process of allocation scheduling (Duran & Rico, 2005).

2.4.1.5 Data structure

Data may be organized many different ways. The logical or mathematical model of a particular organization of data is called a "data structure The choice of data model depends on two considerations:

- It must be rich enough in structure to mirror the actual relationship of data in the real world.
- The structure should be simple enough that one can effectively process data when necessary.

These two considerations are fulfilled by the graph theoretical concepts. The arbitrary relation among data can also be represented by a graph and its matrices ; operations performed on these metrics are further useful for deriving relations and data association and is useful in order to understand how these data may be stored in memory (Manjula, 2012; Williams, 1971).

2.4.1.6 Data mining

Data mining is the process of extracting predictive or hidden data from the large database or in the data warehouses. Graph mining is the main application area of graph theory in data mining. Graph mining represents the relational aspect of data. There are five theoretical based approaches of graph based data mining. They are subgraph categories, subgraph isomorphism, graph invariants, mining measures and solution methods.

2.4.1.7 Operating system

A graph is a data structure of the finite set of pairs, called edges or vertices. Many practical problems can be solved with the help of graph in the field of operating system such as job scheduling and resource allocation problems. For example, graph coloring concept can be applied in job scheduling problems of CPU, jobs are assumed as vertices of the graph, and there will be an edge between two jobs that cannot be executed simultaneously and there will be one to one relationship between the feasible scheduling of graphs (Ahmed, 2012).

2.4.1.8 Website designing

Website designing can be modeled as a graph, where the web pages are represented by vertices and the hyperlinks between them are represented by edges in the graph. This concept is known as a web graph. Which discover the interesting information? Other application areas of graphs are in the web community. Where the vertices represent classes of objects, and each vertex representing one type of objects, and each vertex representing a type of object is connected to every vertex representing another kind of objects. In graph theory, such a graph is called a complete bipartite graph. There are many advantages of using graph representation in website development such as:

- Searching and community discovery.
- Graph representation (directed graph) in website utility evaluation and link structure.
- Finding all connected component and provide easy detection.

2.4.2 Image processing

Image Analysis is the methodology by which information from images is extracted. Image analysis is mainly performed by digital image processing techniques. The image processing techniques can be improved using a graph theoretic approach. The applications of graphs in image processing are:

- To calculate the alignment of the picture as well as find edge boundaries
- Finding mathematical constraints such as entropy by using minimum spanning tree.
- Finding distance transforms of the pixels and calculates the distance between the interior pixels by using shortest path algorithms.

2.4.3 Graphs in Chemistry

Graphs are used in the field of chemistry to model chemical compounds Shirinivas *et al*. (2010). In computational biochemistry, some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modeled in the form of a graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices, (sequences) to eliminate all conflicts. In brief, graph theory has its unique impact in various fields and is growing large nowadays. The subsequent section analyses the applications of graph theory especially in computer science.

Graphs are used to model molecule structures for computer processing Patel and Patel (2013). Here atoms can be considered as vertices of a graph the bonds that connect them are represented as edges between them. This structures are created based on the properties of compounds and are taken for analysis and processing. This can be used to study the structure of molecules and to check similarity level between molecules.

2.4.4Graphs in Biology

A Graph Theory is a very vast subject; it is also extensively used for the analysis in biological networks Patel and Patel (2013). In biology analysis, the number of components of the system and their interactions is distinguished as network, and they are normally represented as graphs where lots of nodes are connected with thousands of vertices (Pavlopoulos *et al*., 2011). Graphs are widely used in following biological analysis; Protein-protein interaction (PPI) networks, Regulatory networks (GRNs), Signal transduction networks, and Metabolic and biochemical networks. If analysis above components then it will be generated the structure network which is similar to one of the graph component in graph theory. Graph isomorphism method can be used for matching two components in the biological analysis. If two graphs are isomorphic to each other than can conclude that the following biological component like protein interaction, biochemical have same molecular property in the biological component. Likewise isomorphism there is subgraph can also be applied for the biological analysis method. If among two graphs one of the graphs is sub graph than in biological analysis the sub graph component formula can be derived from main biological graph component. According to above example, must have knowledge about graph theory then only can understand the concept of biological analysis in the real world.

2.4.5 Graph theory to image segmentation

Graph theory is considered as a most powerful tool in order to explain the algorithms based on image processing, and the theoretical result of graph theory helps to analyze the methods (Dezső *et al*., 2012). In order to segment the image based representation such as a graph, the image is separated into a number of connected components. The graph partitioning is represented as a vertex labeling and graph cut. It is found that these two representations are related to one another, and there is a chance for representing one over the other (Malmberg, 2011). In spite of several years of research, finding a solution for this problem is a difficult task since the segmentation is considered to inherently ill-posed. By considering different segmentation methods, the graph theoretical approach is found to have good features to implement in practical application. It categorizes the image elements mathematically through a structure that is well defined with the formulation for creating an image segmentation problem, and the computation is found to be more effective.

Graph theory is becoming increasingly important as it is applied to different areas of science, mathematics, and technology (Pirzada & Dharwadker, 2007). It is also specifically used in the fields of genomics, communication networks, coding theory, algorithms, computation and detection of diseases. One of the best combinatorial techniques is represented in the graph theory, and it justifies the fundamental results when compared to the other range of mathematics. This type of technique is found to be more cost-effective and efficient way in order to execute the image segmentation. As an important branch of computer vision and image processing, this segmentation and extraction of target objects attracted the attention of researchers Hence the target extraction and image segmentation are used in the field of pattern recognition, artificial intelligence, and computer vision, etc. Future research will be able to give a solution to target extraction and image segmentation. In addition, future research would be able to provide motivation towards the advancement of the pattern recognition, artificial intelligence, and computer vision,etc. (i.e.,) the primary motivation can be displayed. The main role of the theory in terms of computer applications is the improvement of the algorithm based on the graph and numerous algorithms which can solve the problems in the form of graphs.

Mendhule *et al*. (2015) Image processing is a method to convert an image into digital form and perform some operations on it, in order to get an enhanced image or to extract some useful information from it. It is a type of signal dispensation in which input is an image, like video frame or photograph and output, may be image or characteristics associated with that image. Usually, Image Processing system includes treating images as two-dimensional signals while applying already set signal processing methods to them.

Image segmentation defines as the dividing the image through a different subset in which each one of the subsets represents the meaningful part of the image. The image segmentation is an integral part of most of the large problems the quality affects the performance of the entire system. Over the last few decades, there are different types of literature which are published based on the image segmentation process. The most commonly used image segmentation process is edge-base technique, threshold technique, region based technique(Boykov *et al.*, 1999).Based on the approach choice it is difficult to lie the framing knowledge which is prior in to the process of segmentation. The threshold image segmentation takes the decision based upon the pixel information that is been found locally and hence this is more effective in a situation such as level of intensity of the object which falls outside the level of range in the background. Due to the information which is found spatially are avoided by these methods and hence they have unclear boundaries of the region that would generate confusion. Edge based method mainly based on the contour detection. In addition, to this, most of the edge based image segmentation techniques are tend to become weak when joining together and it breaks the contour lines that causes failure in the presence of blurring. In the region-based method, the image is partitioned first for the connected regions by grouping the pixels in the neighborhood that has a similar level of intensity. Then the regions that are adjacent are merged on some conditions such as sharpness or homogeneity of the boundaries region. The preserving connectivity is based on the relaxation-based segmentation method that performs initial boundary shape characteristics in the form of a spline curve, and this can be modified by applying the shrink/expansion operation based on the energy function(Ukunde *et al.*, 2012). In next section will discuss the concept of graph theory in segmentation.

The purpose of image segmentation is to partition an image into meaningful regions with respect to a particular application. Image segmentation is a low-level image processing task that aims at partitioning an image into regions in order that each region groups contiguous pixels sharing similar attributes (intensity, color, etc.). It is a very important process because it is the first step of the image understanding process, and all other steps such as feature extraction, classification, and recognition, depend heavily on its results. Image segmentation is a low-level image processing task in image applications such as machine vision and robot navigation. Recently automated image segmentation techniques with a computer are adopted for improving throughput, reducing cost, diminishing human bias and increasing the intelligence level of the robot. However, today computer-assisted IR image segmentation techniques have not been successfully operated for many reasons. Especially the limited resolution and electronic noise of sensors reduce qualities of images, and the interactive heat environments increase the complexities of identification (Xiaodong Lu & Jun Zhou, 2007). The Segmentation of an image entails the division or separation of the image into regions of the similar attribute. The ultimate aim in a large number of image processing applications is to extract important features from the image data, from which a description, interpretation, or understanding of the scene can be provided by the machine (Karnan & Gopal, 2010).

2.5 Previous studies

2.5.1 Studies related to remote sensing

With the development of remote sensing technology and the improvement of satellite spatial resolution, high-resolution remote sensing images are widely used in various fields (Yanhong *et al.*, 2013). These high-resolution remote sensing images have clear details and rich spatial and texture information. In order to take full advantage of this information, started paying attention to the object-oriented image analysis. For that segmentation technique using graph theory is very efficient. The previous studies based on this are discussed as below.

Mercovich *et al.* (2011)

suggested a modularity clustering technique towards spectral image clustering that does not rely on any typical spectral image processing data models. Rather than utilized image statistics, vector subspaces, or spectral mixing, the modularity technique uses a graph representation of the image to the cluster. Furthermore, they employed, how to arrive at an accurate graph representation of the image. The graph based clustering defines clusters based on nonlinear decision surfaces. Cluster size is not limited unless one is imposed by the user. Modularity is successful at producing a traditional cluster map of image data which was well modeled by traditional methods as well as data where typical models are distorted by clutter. Additionally, the modularity technique produces a variable level of detail cluster-map output and a cluster tree that have additional utility in image analysis. Finally, they concluded as the departure from typical data models and the useful outputs in addition to a simple cluster map make the modularity method a useful technique for automatic clustering.Dezső *et al*. (2012) discussed about the classification of satellite images using the comparison of different graph-based image segmentation methods. In this article four graph-based image segmentation algorithms are compared and evaluated. Namely, the best merge algorithm, tree merge segmentation, minimum mean cut segmentation, and finally normalized cut algorithm. After segmentation, segments are allocated to cover categories along with supervised classification. Then the result of classification is utilized to measure the accuracy of the procedure. The theoretical background and implementation details of each algorithm are described and introduced possible improvements. Their experiments showed that top-down (cut-based) methods show better thematic accuracy than bottom-up (merge-based) algorithms. The best result was provided by the normalized cut based image segmentation. The minimum mean cut algorithm might be sped up with additional heuristic and with more efficient matching implementation, but anyway, its accuracy makes it impractical in remote sensing. In addition, experiments showed that segment-based algorithms become more robust against classification errors with Bhattacharyya classification method. The authors intend to test the methods presented in further areas of Hungary having different cultivation structure/ even in other countries. To content the needs of operational applications, introducing the presented methodology in some other domains is planned as well. Further, they planned to do the research on the examination and comparison of other segmentation methods and segment-based classification procedures.

Yang *et al*. (2015) proposed a image segmentation algorithm for high-resolution remote sensing imagery, which is based on graph theory and fractal net evolution approach (FNEA). For that, an image is modeled as a weighted undirected graph, in which nodes correspond to pixels and edges connect the adjacent pixels. An initial object layer can be obtained proficiently from graph-based segmentation, which runs in time nearly linear in the number of image pixels. Then the proposed FNEA starts with the initial object layer and a pairwise merge of its neighbor object with the objective to minimize the resulting summed heterogeneity. After that, according to the character of different features in the high-resolution remote sensing image, three dissimilar merging criterions for image objects based on spatial and spectral information are adopted. Lastly, compared with the commercial remote sensing software eCognition, the experimental results demonstrated that the efficiency of the algorithm has significantly improved, and the result can maintain good feature boundaries. From the experiment result, found the algorithm is much faster than the original FNEA, which is integrated into the commercial software Cognition. However, this approach needs to focus towards speed up the segmentation algorithm using parallel processing. Further, the texture features need to compute the heterogeneity.

Sirmacek (2011) presented a classification of building facades using graph theory and mean shift segmentation.Such type of automatic classification of building facades has become a field of interest for researchers since it can provide important solutions to the much remote sensing research field. Since urban scene understanding, mobile building recognition systems can be counted as the most important application fields; the Classification results can be also used to increase the level of details in three-dimensional city models which are generated from aerial or satellite imagery. The characterization of stable regions in facades is also necessary for robust indexation and image retrieval. For the study, mean shift segmentation is applied first to the color image and the connected pixels having similar color properties are labeled as one segment. Then using graph theory classified the segments as window, wall, and sky. Then mass centers of the detected segments form the nodes of the graph, and the neighborhood information is obtained through Delaunay triangulation. Next, a graph-cut is by using the graph of the image. Finally, used a probabilistic verification step to compensate shortcomings of graph-cut step.The proposed algorithm is tested on a facade data set which is composed of much different kind of optical facade images which are taken from different places.

Chen *et al*. (2015)proposed a segmentation approach based on graph theory which named self-adaptive mean-shift for high-resolution remote sensing images. This proposed approach could overcome the defects that classic Mean-Shift must determine the fixed bandwidth through trial many times, and could effectively distinguish the difference between different features in the texture rich region. Segmentation experiments were processed with WorldView satellite image. The results show that the proposed method is adaptive, and its speed and precision can satisfy application, so it is a robust automatic segmentation algorithm.

2.5.2 Studies related to image analysis

In this section have discussed the previous studies related to graph based image analysis Further, have identified the issues/ future scope related to these studies and discussed as follows:

Sanfeliu *et al*. (2002) presented several graph-based representations and techniques applied to image modeling, processing, and analysis. They aimed to show how these techniques can be used in practical cases for robot vision, covering the stages from low-level image segmentation to high-level 3D object recognition. Graph techniques permit to represent image objects and scenes in a very natural way, without losing the critical information of each one of the parts that belong to the image object or scene. The graph-based representation and techniques presented here avoid the computational complexity of typical graph problems while offering a good performance in practice. Some experimental results have been shown for image segmentation, perceptual grouping and object recognition.

belong Kale *et al*. (2015) developed an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm for generating weights, regulates the execution, Connectivity Parameter, cutoff, the number of recursions. The First presents a segmentation algorithm within a framework which is independent of the feature used and enhances the correctness and stability with respect to the parameter of different images. In this study image segmentation using graph cut method which finds parameter peak signal to noise ratio, RI, gce, voi. Developing an algorithm to process a distribution of data on graphs is an exciting area. Many biological sensory units are non-uniformly distributed space with spatial distribution often differentiating radically between species. The Medical image analysis creates the development of a graph theory library for the ITK library a necessary and useful addition to accurate and effective processing and analysis of images. This graph cut method is used in medical field for image segmentation which gives the high accuracy. This approach will applicable to various application like data processing, clustering, segmentation of natural images, medical field and so on..

Dikholkar *et al*. (2015) developed an application using isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm for generating weights, regulates the execution, Connectivity Parameter, cutoff, the number of recursions, present some basic background information on graph cuts and discuss major theoretical results, which helped to reveal both strengths and limitations of this surprisingly versatile combinatorial algorithm, further algorithm results are compared with Berkeley's Database on two points PSNR (Peak Signal to Noise Ratio) & RI (Rand Index). The comparative analysis of Berkeley's database is in with two parameters are on better side with good results. This study does not focus on naturally modeled with graphs, for example, segmentation in space-variant architectures supervised or unsupervised learning, 3-dimensional segmentation, and the segmentation/clustering.

Kale *et al*. (2015) intended as an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm for generating weights, regulates the execution, Connectivity Parameter, cutoff, the number of recursions. This study presents some basic background information on graph cuts and discusses major theoretical results, which helped to reveal both strengths and limitations of this surprisingly versatile combinatorial algorithm. After simulation, probable outcomes will be obtained as the weight of all edges. The segmented image will be having high accuracy and high computational efficiency as compared to another algorithm. As it has been shown the application of Graph Theory and its algorithms in Image Processing and especially in the area of Medical Image Analysis makes the development of a Graph Theory library for the ITK library a necessary and useful addition to effective and accurate processing for analysis of images. The implementation is flexible in order to allow it to be applied to varying problems. Further, they planned to design Prim's minimum spanning tree, depth-first search, Dijkstra's shortest path algorithm, and Kruskal's minimum spanning tree algorithm as future work. Also, designs for the graph traits classes will be made more generic and user defined. This way the application of all the graph classes will be truly generic, and graph theory can be applied easily for image analysis.

2.5.3 Studies related to clustering

The previous studies related to clustering based image segmentation are discussed as follows:

Wu and Leahy (1993) proposed a graph theoretic approach for data clustering.Then presented its application to the image segmentation problem. For that, the data to be clustered are represented by an undirected adjacency graph $\mathcal{G}$ with arc capacities assigned to reflect the similarity between the linked vertices. Clustering is achieved by removing arcs of $\mathcal{G}$ to form mutually exclusive sub graphs such that the largest inter-sub graph maximum flow is minimized. For graphs of moderate size (~ 2000 vertices), the optimal solution is obtained through partitioning a flow and cut equivalent tree of $\mathcal{G}$, which can be efficiently constructed using the Gomory and Hu (1961). However for larger graphs this approach is impractical. New theorems for sub graph condensation are derived and are then used to develop a fast algorithm which hierarchically constructs and partitions a partially equivalent tree of much reduced size. This algorithm results in an optimal solution equivalent to that obtained by partitioning the complete equivalent tree and is able to handle very large graphs with several hundred thousand vertices. The new clustering algorithm is applied to the image segmentation problem. The segmentation is achieved by effectively searching for closed contours of edge elements (equivalent to minimum cuts in $\mathcal{G}$), which consist mostly of strong edges, while rejecting contours containing isolated strong edges. This method is able to accurately locate region boundaries and at the same time guarantees the formation of closed edge contours.

Pavan and Pelillo (2003) developed a framework for the image segmentation problem based on a new graph-theoretic formulation of clustering. The approach is motivated by the analogies between the intuitive concept of a cluster and that of a dominant set of vertices, a novel notion that generalizes that of a maximal complete subgraph to edge-weighted graphs. It also establishes a correspondence between dominant sets and the extreme of a quadratic form over the standard simplex, thus allowing the use of continuous optimization method such as replicator dynamics from evolutionary game theory. These systems are attractive coded in a few lines of any high-level programming language, and it can easily be implemented in a parallel network of locally interacting units and offer the advantage of biological plausibility. The present experimental results on real-world images which show the effectiveness of the proposed approach.The framework, is general and can be applied in a variety of computer vision and pattern recognition domains such as for example, texture segmentation, perceptual grouping, and the unsupervised organization of an image database.

Sarsoh *et al*. (2012) proposed a method for classifying of human face images based on the graph theory.This is an effective clustering algorithm which depends on the graphic theory by using the terms and definitions of the graph and the tree. This algorithm was applied to different human face images taken from ORL database, and it gives good clustering results with a small rate of error. For all the experiments, they firstly implement the proposed algorithm by using constant threshold value. Secondly, they extracted adaptive threshold and the adaptive neighborhood for each tested human face image. From that it is obtained the clustering results by using adaptive threshold are better than using a constant threshold. They concluded that It's an automatic algorithm since it does not need to give the number of the resulted clusters a priori. In fact, the obtained results simulate the real structure of the applied data. Using adaptive threshold is better than using constant threshold. It's an effective clustering algorithm since it gives good clustering results with a small rate of error and it took a reasonable execution time for all the experiments. This is a hard clustering algorithm since its clustering results are such that the face images of each person will be in the same tree (cluster).Two types of errors noticed are in few cases is face images for more than one person lie in the same cluster. The face images of some person were partitioned into two clusters.

2.5.4 Studies related to wavelet transform

The existing methods related to wavelet transform with the input as image are discussed as follows:

Lürig *et al*. (1997) proposed a technique using graph algorithms in 3D MRI or CT data with the help of wavelets. The fundamental issue arises from using the graphical algorithms to find structures and assign multidimensional information with the wavelets addressed by the particular segmentation procedure. Through the several interactions of both the concepts that are theoretical through which a report is generated with the graphical attribute and found a mechanism related to the unique range of value that determines the medical features.

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Hammond *et al*. (2011) proposed a novel method for constructing wavelet transforms of functions defined on the vertices of an arbitrary finite weighted graph. Their approach is based on defining scaling using the graph analog of the Fourier domain, namely the spectral decomposition of the discrete graph LaplacianL. The spectral graph wavelets are then formed by localizing this operator by applying it to an indicator function.They explored the localization properties of the wavelets in the limit of fine scales. They also presented a fast Chebyshev polynomial approximation algorithm for computing the transform that avoids the need for diagonalizing L. As a result, proposed wavelet transform is over complete by a factor of J+1 where J is the number of wavelet scales. However, the spectral graph wavelets presented here are not directional.

Mishra *et al*. (2014) presented a comparison between graph method and dual-tree complex wavelet transform used for detecting video shot boundaries. Both the methods are tested in the presence of illuminance effect and motion. This paper showed that the Graph method produces a good result over the other method. Detection of gradual transition and the elimination of disturbances which produced by illumination change or fast object and camera motion are the main challenges to the current shot boundary detection techniques. These disturbances are often wrongly taken as shot boundaries. Therefore, it is a challenging task to develop a method that is not only insensitive to various disturbances but also sensitive enough to capture a shot change. To address these challenges, The gesture features were presented as a task for outlier detection which is useful for the detection of gradual changes in the videos with the help of z-score matrix. They presented Gesture Interpretation is better than color features for detecting hard cuts. Though color performs better for gradual transitions, the spatial domain structure similarity algorithm, which is also invariant to average luminance and contrast, has been utilized on these structure features to detect shot boundaries. To end with, local and adaptive thresholds are used to declare correct shot boundaries. This proposed algorithm is successful in avoiding disturbances due to illumination change and fast motion when the camera follows the object. However, our method is sensitive to unusual cases when the background in consecutive frames change rapidly in addition to the appearance and disappearance of multiple objects in the same scene and using multiple cameras.

Pham *et al*. (2014)this paper proposes a texture-based segmentation method for very high spatial resolution imagery. Indeed, the main objective is to perform a sparse image representation modeled by a graph and then to exploit the wavelet transform on the graph for the purpose of image segmentation. Here, a set of pixels of interest is called representative pixels, and it is first extracted from the image and considered as vertices for constructing a weighted graph. Once the wavelet transform on the graph is generated, their coefficients serve as textural features and will be exploited for unsupervised segmentation. Experimental results show the effectiveness of the proposed method when applied to very high spatial resolution multi-spectral images in terms of good segmentation precision as well as low complexity requirement. However, this study does not focus on the extraction of representative pixels and the definition of vertex description vectors supporting the construction of graph.

2.5.5 Studies related to image segmentation

The previous studies related to image segmentation are discussed as follows:

Felzenszwalb and Huttenlocher (2004) addressed the problem of segmenting an image into regions. It defines a predicate for measuring the evidence for a boundary between two regions using a graph-based representation of the image. Then develop an efficient segmentation algorithm based on this predicate and show that although this algorithm makes acquisitive decisions, it produces segmentations that satisfy global properties. The proposed algorithm to image segmentation using two different kinds of local neighborhoods in constructing the graph, and illustrate the results with both real and synthetic images. The algorithm runs in time nearly linear in the number of graph edges and is also fast in practice. An important characteristic of the method is its ability to preserve detail in low-variability image regions while ignoring detail in high-variability regions.

Skurikhin (n.d.) presented a method for hierarchical image segmentation and feature extraction. This method builds on the combination Canny edge detection and the image Laplacian that is followed by the construction of a hierarchy of segmented images. These images are signified sets of polygonized pixel patches attributed with spectral and structural characteristics. This chain of command forms the basis for object-oriented image analysis. To build a fine level-of-detail illustration of the original image, seed partitions are built upon a triangular mesh composed of irregular sized triangles, whose spatial arrangement is adapted to the image content. This is achieved by building the triangular mesh on the top of the detected spectral breaks that form a network of constraints for the Delaunay triangulation. The polygonized image is signified as a spatial network in the form of a graph with vertices which correspond to the polygonal partitions and graph edges reflecting pairwise partitions relations. This partitioning method is based on the iterative graph contraction using Boruvka's Minimum Spanning Tree algorithm. An important characteristic of the approach is that the agglomeration of partitions is constrained by the detected spectral discontinuities; thus, the shapes of agglomerated partitions are more likely to correspond to the outlines of real-world objects.This proposed method yields good results that can be used by the search for objects of interest across the generated image hierarchy. Further, improves the edge set that can preserve semantically salient, but weak edges might require combination with region-based approaches. This study does not focus the scale-space texture analysis, in particular, adding textures characteristics to the feature vectors describing polygons and their pairwise relations.

Jagannathan (2005) in the context of recognition of free-form objects, which are characterized by range sensor generated 3D point clouds, this dissertation addresses two fundamental issues (1) segmentation of surface meshes constructed over the input point clouds, (2) determination of correspondence between the Scene and Model point clouds. Many existing recognition systems require uniform sampling of the Model and the Scene, or they assume that these point clouds overlap. This dissertation describes the solutions to the segmentation and correspondence problems without resorting to any of these restrictive assumptions. Mesh segmentation is an important step in deriving an efficient representation of the underlying object and is challenging due to noisy input data. In the proposed approach, curvedness, which is a rotation and translation invariant shape descriptor, is computed at every vertex in the input mesh. Iterative graph dilation and morphological filtering the outlier curvedness values result in multiple disjoint sub-meshes corresponding to the physical parts of the underlying object. Results indicate that the algorithm compares well with the existing state-of-the-art approaches, and it provides robust segmentations in the presence of noise.

Robles-Kelly (2005) this study describe the use of graph-spectral techniques and their relationship to Riemannian geometry for the purposes of segmentation and grouping. Here the problem of segmenting a set of tokens as that of partitioning the set of nodes in a graph whose edge weights are given by the geodesic distances between points in a manifold. To do this, it commences by explaining the relationship between the graph Laplacian, the incidence mapping of the graph and a Gram matrix of scalar products. It permits the recovery of the embedding coordinates in a closed form and opens up the possibility of improving the segmentation results by modifying the metric of the space in which the manifold is defined. The set of embedding coordinates finds the partition of the embedding space which maximizes both, the inter-cluster distance and the intra-cluster affinity. The utility of the method for purposes of grouping is illustrated on a set of shape silhouettes.

Harchaoui and Bach (2007) proposed kernels between their respective segmentation graphs. The kernels are based on the soft matching of subtree-patterns of the respective graphs, leveraging the natural structure of images while remaining robust to the associated segmentation process uncertainty. Actually, the output from morphological segmentation is often represented by a labeled graph, each vertex corresponding to a segmented region, with edges joining neighboring regions. Though, such image representations have mostly remained underused for learning tasks, partly because of the observed instability of the segmentation process and the inherent hardness of inaccurate graph matching with uncertain graphs. Our kernels count common virtual structures amongst images, which enables to perform efficiently supervised classification of natural images with a support vector machine. Furthermore, the kernel machinery permits us to take advantage of recent advances in kernel-based learning: i) semi-supervised learning method it reduces the required number of labeled images while ii) multiple kernels learning algorithms it efficiently selects the most relevant similarity measures between images within our family.

Gilboa and Osher (2007)proposed a nonlocal quadratic function for image feature extraction. Here the weights are based on image features and represent the affinity between different pixels in the image. By suggesting different formulas for the weights, one can generalize many local and nonlocal linear denoising algorithms with the nonlocal means filter and the bilateral filter. In this proposed framework easily show that continuous iterations of the generalized filter observe certain global characteristics and converge to a constant solution. The linear operator related with the Euler-Lagrange equation of the functional is narrowly related to the graph Laplacian. Thus, interpret the steepest descent for minimizing the functional of the nonlocal diffusion process. This formulation allows a convenient framework for nonlocal variational minimizations, including variation denoising, Bregman iterations, and the recently proposed inverse-scale-space. It is also demonstrated how the steepest descent flow can be used for segmentation. Following kernel-based methods in machine learning, the generalized diffusion process is used to propagate sporadic initial user's information to the entire image. The classical variational segmentation methods not explicitly based on a curve length energy and thus can cope well with highly non-convex shapes and corners. Reasonable robustness to noise is still achieved.

Narayan *et al*. (2008) proposed an improvement of the graph based image segmentation methods already described in the literature. Graph based image segmentation techniques are considered to be one of the most efficient segmentation techniques which are mainly used as time & space efficient methods for real-time applications.They proposed a new segmentation method with the use of a weighted Euclidean distance to calculate the edge weight which is the key element in building the graph. This help to select more prominent edges in the graph. The experimental results showed the improvement in the segmentation quality as compared to the methods that already exist, with a slight compromise in efficiency.

Yin *et al*. (2009)presented a novel graph construction method and demonstrated its usage in a broad range of applications starting from a relatively simple single-surface segmentation and ranging to very complex multi-surface multi-object graph based image segmentation. Inspired by the properties of electric field direction lines, the proposed method for graph construction is inherently applicable to n-D problems. In general, the electric field direction lines are used for graph “column” construction. As such, their method is robust with respect to the initial surface shape and the graph structure is easy to compute. When applied to cross-surface mapping, our approach can generate one-to-one and every-to-every vertex correspondent pairs between the regions of mutual interaction, which is a substantially better solution compared with other surface mapping techniques currently used for multi-object graph-based image segmentation.

Evaluated images using graph theory for the remote sensing image is segmented by the Ncut algorithm in graph theory.Then the technology of the optimization is introduced into the the remote sensing image segmentation. The performance analysis and evaluation of the remote sensing image segmentation have the theoretical basis. And the effective adaptive analysis model has been achieved. The evaluation of adaptive analysis model of the remote sensing image segmentation is mainly due to estimation of the time complexity. Because many adaptive analysis model structure of the remote sensing image segmentation were put forward, it is not feasible that the efficiency of the analysis model is measured in actual images and then comparation is done. Therefore, the optimum technology is introduced into the adaptive analysis model of the remote sensing image segmentation. It makes the performance evaluation of the adaptive analysis model of the remote sensing image segmentation has a certain theoretical basis. The methods have certain practical value for the research and development of the segmentation data of the remote sensing image.

Janakiraman and Chandra Mouli (2010) presented an algorithm for image segmentation problem using the concepts of Euler graphs in graph theory. By treating the image as an undirected weighted non-planar finite graph (G), image segmentation is performed as graph partitioning problem. The proposed method locates region boundaries or clusters and runs in polynomial time. Subjective comparison and objective evaluation showed the efficiency of the proposed approach in different image domains.The procedures discussed run in polynomial time. The MST and cycles method performs better compared to Euler Graph method in terms of precision, recall, and F-measures.

Ukunde *et al*. (2012) presented a performance evaluation of image segmentation using histogram and graph theory. Within the recently available segmentation methods, graph theory based methods had efficient performance in practical applications. Those techniques can explicitly organize the image objects into mathematical structures, making the computation of image segmentation problem more flexible and efficient. So that This research work used concepts of both histogram and graph theory for image segmentation. For obtaining the performance the PSNR and time taken for image, segmentation have been used as a comparison parameter. From the results, it can be seen that histogram based segmentation technique requires small segmentation time in comparison to graph theory based segmentation techniques but suffers from lower PSNR values. In addition, the use of Haar and DB2 wavelet for graph-based segmentation has good PSNR and low segmentation time.

Alpert *et al*. (2012) presented a bottom-up aggregation approach for image segmentation. At the start with an image, execute a sequence of steps in which pixels are gradually merged to produce larger and larger regions. In each step, consider pairs of adjacent regions and provide a probability measure to assess whether or not they should be included in the same segment. The proposed probabilistic formulation is intensity and texture distributions in a local area around each region. It further joins priors based on the geometry of the regions. To end with, posteriors based on intensity and texture cues are combined using a mixture of experts formulation. This approach is integrated into a graph coarsening scheme and providing a complete hierarchical segmentation of the image. The algorithm complexity is linear in the number of the image pixels, and it requires almost no user-tuned parameters. In addition, this studies attempting to avoid human semantic considerations that are carried out of scope for segmentation algorithms. Using this novel evaluation scheme, test the method and provide a comparison to several existing segmentation algorithms.

Presented a semi-supervised strategy to deal with the issue of image segmentation. Each image is first segmented coarsely and represented as a graph model. Then, a semi-supervised algorithm is utilized to estimate the relevance between labeled nodes and unlabeled nodes to construct a relevance matrix. Finally, a normalized cut criterion is utilized to segment images into meaningful units. The experimental results conducted on Berkeley image databases and MSRC image databases demonstrate the effectiveness of the proposed strategy.

Soltanpoor *et al*. (2013) presented a graph-based image segmentation using the imperialist competitive algorithm. In this article, to resolve the issue of image segmentation, input image converts into a graph after initial pre-processing. The obtained graph is then partitioned with the imperialist competitive algorithm, and a number of crossing edges optimize through the graph.By comparing the results of this algorithm with the colonial competitive algorithm, more time is spent. But the manufacturer has optimized the output image segmentation is more applicable. In this segmentation method 4-connected scheme is used to create image graph. In these methods, image converts to a binary mode that it can be considered as a colored image. The better results in graph partitioning and its usages in image segmentation can be anticipated by composing ICA and chaos theory or other optimizing algorithms.

Casaca *et al*. (2014) presented seed-based image segmentation methods. In this study, they presented a novel framework for seed-based image segmentation that is mathematically simple, easy to implement and guaranteed to produce an exclusive solution. Furthermore, the formulation holds an anisotropic behavior and pixels sharing similar attributes are kept closer to each other while big jumps are naturally imposed on the boundary between image regions, thus ensuring better fitting on object boundaries. They showed that the proposed framework outperformed state-of-the-art techniques in terms of quantitative quality metrics as well as qualitative visual results.

Casaca (2014) presented a spectral-based segmentation algorithm combined image decomposition, similarity metrics, and spectral graph theory into a concise and powerful framework. Image decomposition is implemented to split the input image into texture and cartoon components. Then, an affinity graph is formed, and weights are assigned to the edges of the graph according to a gradient-based inner-product function. By using the eigenstructure of the affinity graph, the image is divided through the spectral cut off the underlying graph. Furthermore, the image separating can be improved by changing the graph weights by sketching interactively. Visual and numerical evaluations were conducted against representative spectral-based segmentation techniques using boundary and partition quality measures in the well-known BSDS dataset. Advantages of using this method are, besides its simple mathematical formulation, Laplacian Coordinates is easy to implement, guarantees a unique solution and outperforms existing methods with respect to well-established quantitative measures popularly used in the context of image segmentation. The laplacian Coordinates also embraces high accuracy in terms of image boundary fitting capability. All those properties render Laplacian Coordinates an interesting and compelling seed-based image segmentation technique.

Fernández-Mota *et al*. (2014) proposed an algorithm to formulates the line segmentation to finding the central path in the area between two consecutive lines. It's resolved by graph traversal problem. A graph is made using the skeleton of the image. Then, a path-finding algorithm is used to find the optimum path between text lines. This proposed method evaluated on a comprehensive dataset consisting of five databases: ICDAR2009, UMD, ICDAR2013, the George Washington and the Barcelona Marriages Database. This method performs state of the art considering the different types and difficulties of the benchmarking data.

Huang *et al*. (2014) proposed a parameter automatically optimized robust graph-based image segmentation method and performed on breast ultrasound images. A PSO algorithm is incorporated with the RGB method to achieve optimal / approximately optimal parameters. Experimental results have shown that the proposed technique can more accurately segment lesions from ultrasound images compared to the RGB and two conventional region-based methods.Segmentation of medical images is a predictable image processing step for computer-aided diagnosis. Due to the complex acoustic inferences and artifacts, the extraction of breast lesions in ultrasound images remains a challenge. While there have been many segmentation techniques proposed, the performance often varies with different image data that is leading to poor adaptability in real applications. Intelligent computing techniques for adaptively learning the boundaries of image objects are preferred. This study focuses on optimization of a previously documented method called robust graph-based (RGB) segmentation algorithm to extract breast tumors in ultrasound images more adaptively and accurately.The experimental results show the significantly improves the performance of RGB and outperforms of other two conventionally used region based methods. It can be expected that the proposed method will be much capable of extracting lessons from BUS images in various clinical practices.

Parihar and Thakur (2014) present the image segmentation approach based on graph theory and threshold. Among the various segmentation approaches, the graph theoretic approaches in image segmentation make the formulation of the problem more flexible and the computation more resourceful. The segmentation problem is modeled in terms of partitioning a graph into several sub-graphs; such that each of them represents a meaningful region in the image. The segmentation problem is then solved in a spatially discrete space by the well-organized tools from graph theory. After the review of the previous method, the problem is formulated regarding graph representation of image and threshold function. The boundaries between the regions are determined as per the segmentation criteria, and the segmented regions are labeled with random colors. In this proposed method the image is preprocessed by discrete wavelet transform and coherence filter before graph segmentation. The results carried out on a number of natural images taken from Berkeley Database as well as artificial images from online resources. The experiments are accomplished by using the wavelets of Haar, DB2, DB4, DB6 and DB8. The results are evaluated and compared by using the performance evaluation parameters like execution time, Peak Signal to Noise Ratio, Performance Ratio, Precision, and Recall. This study does not carry out the experimentations to other families of wavelets for preprocessing the image before segmentation and observe the results.

Li and Zhu (2015)presented a study to improve the performance of image segmentation. For that segmentation algorithm based on the graph theory is used. This study first segmented the image into different regions and mapped those regions into a graph structure. then, a relevance matrix is formulated by using semi-supervised learning; At last, by normalized segmentation rules, some regions are further refined into perceptive regions. These experimental results are conducted on Berkeley image database, and MSRC image dataset demonstrated the most competitive performance of the proposed image theory segmentation method compared with traditional algorithms.

Haxhimusa and Kropatsch (2004) introduced a method to build a hierarchy of partitions of an image by comparing in a pairwise manner the difference along the boundary of two components relative to the differences of components' internal differences. Even though the algorithm takes simple greedy decisions locally, it produces perceptually important partitions in a bottom-up ’stimulus-driven’ way based only on local differences. It was shown that the algorithm could handle large variation and gradient intensity in images. Since our framework is general enough, can use RAGs of any over-segmented image and build the hierarchy of partitions. External knowledge can help in a top-down segmentation technique. A drawback is that the maximum and minimum criterion are very sensitive to noise, although in practice it has a small impact. Other criteria like median would lead to an NP-complete algorithm. Our future work is to define different comparison functions which will prefer learned regions of specific shapes.

2.5.6 Studies related to graph cut \ Normalized cut segmentation

The existing method related to graph cut and\or normalized cut segmentation method are discussed as follows:

Kapade *et al*. (2014)proposed an enhanced graph based normalized cut method for extracting global impression and consistencies in the image. This proposed a technique to add flexibility of original recursive normalized two-way cut method which was further extended to other graph-based methods. The results show that the proposed technique improves segmentation quality as well as requires lesser computational time than the regular normalized cut method. It shows that multi-scale and watershed segmentation methods consume less computational power, and their performance is almost same for both for all the images considered and it is better than that of computed by. It also indicates that the time complexity for pixel affinity and recursive two-way cut methods is sensitive to the image. The graph-based methods generally perform segmentation on the basis of local properties of the image. For segmenting the images in some applications where detailed extraction of features is necessary, consideration of global impression along local properties is inevitable.This proposed an enhanced technique which allows considering both, local as well as global features during normalized cut based segmentation to meet the requirement of precise segmentation. This was achieved by correlating the feature values around neighboring pixels for determining weights of edges of the graph. This creates strong weight connections between the identical neighboring pixels in the affinity matrix resulting better segmentation quality with linear complexity. The result shows that the final score of multiscale graph decomposition is superior to the score obtained by other methods and even better than that of combining hierarchical multiscale graph decomposition. The technique also has lesser computational time complexity.

Basavaprasad and Ravi (2014) discussed the comparative study on the classification of image segmentation methods with a focus on graph based techniques. From this comparison study, they concluded that there is no universal segmentation method that executed for all types of images, but a number of techniques do healthier than others for particular types of images representative better performance can be attained by selecting appropriate algorithm or combination of suitable techniques.

Nalina and Muthukannan (2013) the main goal of this study is to review the high quality of image segmentation with stability and improved speed. In this study to segment the image using three different graph-based segmentation algorithms. These are Isoperimetric Segmentation Normalized Cut Segmentation and Spectral Segmentation. Apply these algorithms on the image and find out segmentation result. Using the segmentation results the performance will be analyzed with speed and stability. To determine the stability of image by adding the Additive Noise, Multiplicative Noise, Shot Noise. The original image segmented by Spectral segmentation, Normalized cut segmentation, Isoperimetric segmentation. Using this segmentation result, the performance was analyzed with stability and speed. The segmentation time of Spectral segmentation is 2.6877 seconds, Ncuts Segmentation is 5.4910 seconds, and the Isoperimetric segmentation is 0.4931 seconds. The isoperimetric segmentation has very less time to segment the image compare to spectral and N-cuts segmentation. Adding three different Noises to Images, the Isoperimetric Algorithm is more stable compared to N-cuts segmentation.

Liu *et al.* (2010) presented an Image segmentation based on the normalized cut framework. The experiment results showed that their methods could achieve the image segmentation purpose. For simple images with just a little texture inside, the result is quite good, and this method could also perform well on natural and landscape images. The above tests are based on the luminance or RGB-based similarity measurement. When testing this algorithm on the texture images with the same similarity measurement, they found the result become worse. And the main reason is that the pixel value variance is really large in the texture region, which means pixels in the same texture region may have small similarities and are segmented into different groups. Furthermore, in future, they planned to improve the stability of the program. For that code of the function are going to modify. Further, a better adaptive algorithm for segmentation is going to implement. They also were planning to generate the post-processing mechanism for region merging. They will write code using merge groups with the same texture as a single group.

Haxhimusa *et al*. (2006) presented an Evaluation of Hierarchical Graph-based Segmentation. This study evaluated segmentation results of two mostly used graph-based methods: the normalized cut (N-CutSeg) and the minimum spanning tree based (MSTBoruSeg), and compared them with human segmentations. Note that the NCutSeg uses an approximation algorithm to produce segmentation results, whereas MSTBoruSeg a deterministic process. The evaluation is done by discrepancy measures that do not penalize segmentations that are coarser or more refined in certain regions. This study uses only grayscale images to evaluate the quality of results on one single feature. Using real world images, two hierarchical graph-based segmentation methods are evaluated with respect to segmentations produced by humans. Global and local consistency measures do not show big differences between the two representative methods although human visual inspection of the results show advantages for one method. To a definite extent subjective impression is captured by the new criteria of ‘region size variation.' Two segmentation methods did not prove to be as efficient as the humans but showed that, for both the error measure results are concentrated in the lower half of the output domain and that the mean of the GCE measure, which is stronger than LCE, is for both around the value of 0.25. Thus, both of the methods perform similarly if compared with the consistency measures LCE and GCE. In the experiment with region sizes, show that humans have the biggest variation of the produced region sizes, followed by MSTBoruSeg and NCutSeg.

KeucheI and Schnörr (2003) presented the application of graph theoretic methods to unsupervised image partitioning a very active field of research recently. For weighted graphs encoding the (dis)similarity structure of locally extracted image features, unsupervised segmentations of images into coherent structures can be computed in terms of external cuts of the underlying graphs. To overcome this issues, the study focuses on the normalized cut criterion and a related recent convex approach based on semidefinite programming. As both methods soon become computationally demanding with increasing graph size, an important question is how the computations can be accelerated. To this end, study an SVD approximation method in this paper which has been introduced in a different clustering context. This method is based on probabilistic sampling, to both segmentation approaches and compares it with the Nystrom extension suggested for the normalized cut. Numerical results confirm that by means of the sampling-based SVD approximation technique, reliable segmentations can be computed with a fraction of the original computational cost.

Peng *et al*. (2013) proposed a systematic survey of the graph which is made by theoretical methods for image segmentation hence the problem is modeled in terms of separating a graph into several sub-graphs such that each of them signifies a meaningful object of interest in the image. These methods are classified into five classes under a uniform notation. These are minimal spanning tree based methods; graph cut based methods with cost functions, graph cut based methods on Markov random field models, and the shortest path based methods and the other methods that do not belong to any of these classes. The quantitative evaluation method is carried by using five indices. Those are Normalized Probabilistic Rand index, Probabilistic Rand index, Global Consistency Error, Variation of Information and Boundary Displacement Error– on some representative automatic and interactive segmentation methods. Further study can refer to versatile graph-based algorithms for a wide range of practical applications. The choice of which method to use is often application specific, and argue that the successful ones should utilize appropriate graph models and guarantee good properties of the segments.

Ting *et al*. (2013) addressed the issue of vascular ultrasound image segmentation and proposed a novel ultrasonic vascular location and detection method. It contributed in several aspects: Firstly using mean shift segmentation algorithm to obtain the initial segmentation results of vascular images; Secondly new data item and smooth item of the graph cut energy function was constructed based on the MRFmodel, and then put forward swap and an expansion ideas to optimize segmentation results, subsequently locate the lumen and vessel wall in vascular images. Finally, it compared with the manually tagging results, and applying edge correlation coefficients and variance to verify the validity of our algorithm; experimental results show that our algorithm can efficiently combine the advantages of mean shift and graph-cut algorithm and achieve better segmentation results. Furthermore this study need to concentrate two aspects, firstly how to accurately and effectively locate the ultrasound vascular image edge when acquisition effect of ultrasound image are not well, secondly when the amount of medical image data is too large, how to improve calculation speed of maximum flow methods and enhance the graph-cut real-time performance for better adapt for the division of ultrasound images.

Simayijiang and Grimm (2016) presented a study to find graph cut methods for segmenting images and investigated how they perform in practice. Segmentation based on graph cuts works is effective for some images, but for some images, it is very difficult.In their experiments, they mainly used gray level images. When the energy function based on only data term is low, the result has more noise. But the energy function based on both regularization term and data term is high the result has few edges. That means the segmentation based on the gradient of an image needed more detailed user seeds when boundaries of the object don't differ clearly enough from the edges in the background.

Prasad (2016) presented an explanation of graph cuts for image segmentation. This study conducted a survey of various graphic theory methods and found the normalized cut is efficient. "Normalized cuts" is proposed which focuses on extracting the global impression of an image rather than focusing on local features and their consistencies in the image data. This norm measures both the total dissimilarity between the different groups as well as the total similarity within the groups. This study showed that efficient computational technique based on a generalized eigenvalue problem could be used to optimize this criterion.

2.5.7 Studies related to Image Feature Extraction

The previous studies related to extraction of image feature are discussed as follows:

Huang *et al*. (2015) propose a fuzzy local discriminant embedding (FLDE) algorithm based on the unsupervised discriminant projection criterion and fuzzy set theory for image feature extraction and recognition. In this method, a membership degree matrix is firstly calculated using the fuzzy k-nearest neighbor (FKNN) algorithm, and then the membership degree and the label information are incorporated into the definition of the weighted matrices to get the fuzzy local scatter, and fuzzy nonlocal scatter. After characterizing the fuzzy nonlocal scatter and the fuzzy local scatter, a concise feature extraction criterion is derived via maximizing the ratio between them. Experimental results on the ORL, FERET, and CMU PIE face databases show the effectiveness of the proposed method. However, there are still some drawbacks to being considered like a time consuming, need to achieve the optimal results in practical applications remains a problem. Also, try to improve the objective function with kernel trick in nonlinear case.

Du *et al*. (2015) propose a novel Schatten p-norm-based two-dimensional principal component analysis (2DPCA) method, for image feature extraction. Different from the conventional 2DPCA that is based on Frobenius-norm, 2DPCA-Sp learns an optimal projection matrix by maximizing the total scatter criterion based on Schatten p-norm in the low-dimensional feature space. Also proposed an iterative algorithm to solve the optimization problem of 2DPCA-Sp with $0 < p < 1$, which is simple, effective, and easy to implement. Experimental results on several popular image databases show that 2DPCA-Sp with $0 < p < 1$ is robust to impact factors of images. However, this study needs to focus towards improving the computational performance of the proposed 2DPCA-Sp method.

Fukuma *et al*. (2016) discussed nucleus segmentation method, feature descriptors and disease stage classification for Glioma histopathology images. And, the feature descriptors defined in the literature were applied to Glioma images, and statistical significance of them w discussed too. n 98.6% of images were classified correctly using SVM which was consist of Object-Level features, and 82.1% of images were classified correctly using SVM which was consist of Spatial-Arrangement features. From the experimental results, more than 99.8% of images were classified correctly using Random Forests in the case of Object-Level features, and more than 86.1% of images were classified correctly using RF (Random Forests) in the case of Spatial-Arrangement features. From these, the author concluded that the Object-Level features are better than Spatial-Arrangement features to classify the progression of Glioma image in TCGA (m The Cancer Genome Atlas) Data. However, this study needs to focus towards o discover disease subtypes using feature matrices and confirm the relationship of the disease stage and gene expression available within the TCGA data.

Li *et al*. (2013) proposed a novel feature extraction method called ordinal regularized manifold feature extraction (ORMFE) for image ranking. It is based on the observation that most of the feature extraction methods only preserve the local manifold structure of data, but ignore the ordinal information among data groups of different ranking levels, so we aim at preserving both local manifold structure and ordinal information in the low-dimensional subspace, where a ranking model can be learned effectively and efficiently. Extensive Experiments on two benchmarks demonstrated the power of the proposed method compared to several related works.

Wang *et al*. (2014) investigated four representative feature extraction algorithms, color-texture codebook (CT), SIFT codebook, HMAX, and convolutional networks (ConvNet). Comprehensive experiments were conducted that revealed differences between these algorithms. From the experimental results reveal that both training data size and dataset category composition can affect the results of algorithm comparisons. Extreme care must be taken by researchers to avoid such pitfalls to ensure the reporting of reliable results. Finally, we devised and studied a fusion algorithm based on confusion matrices to harvest synergies between these four algorithms. However, this study needs to focus, formally formulating the problem and comparing different design considerations.

2.5.8 Studies related to medical application

The existing methods based on the medical application are discussed as follows:

Chen *et al*. (2006) proposed an approach for the segmentation of lung fields in the severe acute respiratory syndrome (SARS) infected radiographic images, which is the first step towards a computer-aided diagnosis system. To overcome the segmentation difficulty of SARS in the lung images, their algorithm first used morphological operations to obtain the initial estimation of the regions where the lung boundaries lie in and then applied a new graph-based optimization method to find the interested regions. The theoretical analysis showed that their approach is resistant to boundary noise, discontinuity, and large patches that affect the boundary search. Experimental results are given to demonstrate the good performance of this algorithm.

Elmasry *et al*. (2012) presented an approach for segmentation of CT scan liver images using normalized cuts graph partitioning approach. Human-based segmentation of liver computerized tomography (CT) images is very time-consuming, so it is desired to develop a computer-based approach for the analysis of liver CT images that can exactly segment the liver without any human intervention. To evaluate the performance of new approach, They presented tests on different liver CT images. The experimental results obtained showed that the overall accuracy offered by the employed normalized cuts technique is high compared to the well-known K-means segmentation approach. All the results were obtained using two measures that highlight segmentation accuracy to assess the strength of normalized cuts algorithm for segmenting the affected part in the liver. Those two measures are time complexity and segmentation accuracy. In common, the normalized cuts algorithm reached high accuracy, however with the cost of high time complexity. Even though this method is efficient, the accuracy can be increased by using more features, additional to the colon feature used in this paper, such as shape and texture. They also focussed on utilizing multi-objective concept via presenting more features by using other segmentation methods such as k-means clustering algorithm for various medical images. Now the computer assisted system are used in the diagnosis of liver fibrosis; In addition, will investigate an intelligent diagnosis system for diagnosing features derived from the computer tomography images of the liver in Chronic Hepatitis C.

Kapade *et al*. (2014) presented theoretical and practical improvements in image segmentation based on graph theory in the medical field. Their study is based on the use of the region adjacency graph produced by the watershed transform from mathematical morphology. The combination of morphological and graph cuts segmentation permits to speed up and define new classes of energy functions that can be minimized using graph cuts. They also studied properties of minimal spanning trees, shortest paths trees, and minimal cuts, and utilized these notions for image segmentation purposes. This help to formulate the possibilities and limitations of each technique. This study helps to minimize the large variety of energy problems. The use of region graphs gives promising results and can potentially become a leading method for interactive medical image segmentation.

Srinivasan *et al*. (2014) presented an automatic approach for segmenting retinal layers in spectral domain OCT images using sparsity-based denoising, support vector machines, graph theory, and dynamic programming (S-GTDP). Results showed that this method accurately segments all present retinal layer boundaries, which can range from seven to ten, in wild-type and rhodopsin knockout mice as compared to manual segmentation and has a more accurate performance as compared to the commercial automated segmentation software. This proposed two-step approach is the simplest case of a general framework for the automated segmentation of retinal boundaries from eyes with different anatomic and pathologic features. In this framework, the first SVM-based step detected the specific pathology and selected the appropriate algorithm for the data set in hand. The second step utilized any of the layer segmentation algorithms developed in the past few years. The future work will address the most challenging case of segmenting retinal layers in human eyes with multiple types of pathology from different diseases such as diabetic retinopathy, macular hole, and age-related macular degeneration.

2.6 Gaps Identified

After analyzing all the above studies, several gaps are identified. These are discussed as follows.Lürig *et al*. (1997) presented graph algorithms to find important structures and to assign multidimensional information to these structures with the help of wavelets, but they also needed to find mechanisms to determine ranges of characteristic values for interesting medical features. Pavan and Pelillo (2003) presented a method in general and it does not prove this study can apply in a variety of computer vision and pattern recognition domains such as texture segmentation, perceptual grouping, and the unsupervised organization of an image database. Liu *et al*. (2010)require a post-processing mechanism for region merging Skurikhin (n.d.)needed improvement of the edge set that can preserve semantically salient but weak edges might require combination with region-based approaches. In addition, will also need scale-space texture analysis; in particular, adding textures characteristics to the feature vectors describing polygons and their pairwise relations. Sarsoh *et al*. (2012) presented a hard clustering algorithm since its clustering results are such that the face images of each person will be in the same tree (cluster).This study faced two issues one is, in few cases face images for more than one person are lied in the same cluster and the second one is, face images of some person were partitioned into two clusters. The future study must be conducted to remove this problem.

Elmasry *et al*.(2012) implemented segmentation using only color features. For better accuracy additional features such as shape, texture is also needed to be used. This study gave only a concept of segmentation of live CT images. Peng *et al*. (2013) conducted a survey of graph theoretical methods for image segmentation. However, the study fails to provide versatile graph-based algorithms for a wide range of practical applications. Soltanpoor *et al*. (2013) used a 4-connected scheme to create image graph and does not carry the 8-connected on the r-connected scheme. Pham *et al*. (2014) method needed to improve especially for the extraction of representative pixels and the definition of vertex description vectors supporting the construction of graph. This proposed method does not carry the other types of VHR images .Mishra *et al*. (2014) the method is sensitive to unusual cases when the background in consecutive frames changes rapidly in addition to the appearance and disappearance of multiple objects in the same scene and using multiple cameras. Srinivasan *et al*. (2014) presented an approach for segmenting retinal layers. This study does not concentrate the segmentation of retinal layers in human eyes with multiple types of pathology from different diseases such as diabetic retinopathy, macular hole, and age-related macular degeneration also needed to analyze. Yang *et al*. (2015)this study does not focus on two concerns; one is parallel processing to speed up the segmentation algorithm. The second is taking into account the texture features to compute the heterogeneity. Kale *et al*. (2015) an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm. However, it fails to define function classes for Prim's minimum spanning tree, depth-first search, Dijkstra's shortest path algorithm, and Kruskal's minimum spanning tree. In addition need to designs for the graph traits classes will be made more generic and user defined. This way the application of all the graph classes will be truly generic, and graph theory can be applied easily for image analysis.Dikholkar *et al*. (2015) analysis of different parameters used in the algorithm for generating weights regulates the execution, Connectivity Parameter, cutoff, the number of recursions. However, it fails to provide applications to segmentation in space-variant architectures, supervised or unsupervised learning, 3-dimensional segmentation, and the segmentation/clustering of other areas that can be naturally modeled with graphs. However, Due to that NCUT method using image pixel for segmentation, there are exponential numbers of possible partitions of the graph. As a result, it is computationally expensive to find the optimal partition(Zhao, 2015). Graph cut optimization: The limitation of the Graph cut optimization approach is that it requires the number of partitions to be provided by users and hence cannot fully automatically segment an image. How to automatically determine the proper partition number for different images will be studied in the future. In addition, we will also explore the possibility of incorporating the proposed recursive calculation scheme into other information entropy methods (Yin *et al*., 2014).

2.9 Summary

In this section, divided the review of literature into four section. First, briefly discussed the history and definition of graph theory. Second, discussed the application of graph theory to the various field like computer vision, chemistry, biology, remote sensing and so on. Third, discussed the previous studies related to image segmentation particularly using graph theory. Finally, identified the problem or issues of existing method.One of the advantages to formulating the segmentation on a graph is that it might not require discretization by virtue of purely combinatorial operators and thus incur no discretization errors.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

Fast growing field of image processing has put many challenges for mathematicians. It includes researching the ways to improve the efficiency and accuracy in the practical applications by processing the images correctly. Digital image processing opened new avenues for interdisciplinary research for tackling such challenges and provided new directions for research. Today, it has been used for reliable personal identification and authentication techniques such as biometrical systems, medical applications, remote sensing and in many others. Image segmentation is the first stage in any attempt to analyze or interpret an image automatically which bridges the gap between low-level and high-level image processing. The graph theoretical approaches organize the image elements into mathematically sound structures and make the formulation of the problem more flexible and computationally efficient towards improving the performance of graph partitioning algorithms. The main aim of this research is to analyze the applications of Discrete Mathematics in image processing, particular the Applications of Graph Theory in Image Segmentation.

The purposes of this chapter are to,

- describe the research methodology,
- explain the proposed design,
- describe the procedure used in designing and collecting the data,
- Provide a detailed description of graph-based approach
- development of efficient graph based segmentation scheme for performance improvement

3.2 Proposed method

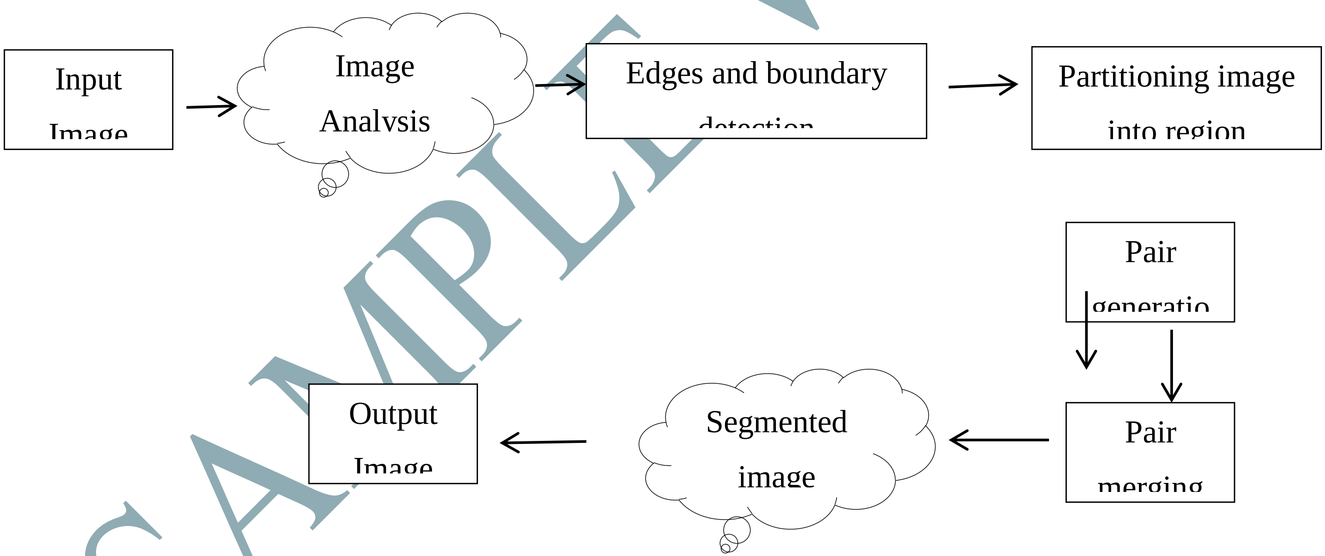
In this research proposed a novel color spatial clustering with consensus region merging for segmenting the image in an effective manner. Here the It cluster and predicate was based on measuring the dissimilarity between pixels along the boundary of two regions. By using this way, able to simplify the representation of an image into more significant and easier by applying graph theory.

This study broadly divided into three sections. Firstly, analyze and implement the existing traditional approach such as minimal spanning trees, normalized cuts, Euler graph, and Iterated Graph cut, OTSU thresholding, K-means segmentation, Split-and-merge methods, and Fuzzy Clustering method. Second, proposed novel approach and implemented using Matlab simulation software. Thirdly, compares efficiency and effectiveness of traditional graph-based image segmentation algorithms with proposed method using medical images like MRI, X-ray, and CT scan images.

3.2.1 System architecture

The system architecture consists of an image as the input, and the image undergoes the process of image analysis. Then partitioned the image and calculated the merging region. Finally, calculate the mutual information by applying the above-proposed algorithm for the images.

Figure 1: Proposed system architecture



The cluster and predicate were based on measuring the dissimilarity between pixels along the boundary of two regions. The merging predicate consists of: (a) estimating for each channel, independently of the others; (b) evaluating the eligibility of merging adjacent nodes and (c) authenticating the consistency of the merged nodes.

Let G be an undirected graph and let $v_i \in V$ -the set of all nodes corresponding to image elements, $e_i \in E$ be the set of edges connecting to the pairs of neighboring nodes in G.

Let $e_i \in E$ be the edge connecting the vertices $v_i, v_j \in V$, for every $i, j \in I$ and $i, j \leq n$. Here $d(e_i) = w_i$ and w_i is used to measure the dissimilarity of the two nodes connected by that edge e_i.

The Predicate is defined as,

$$(P_a, P_b) = \begin{cases} true & if\,(v_i, v_j) < \frac{255}{2}\log\max(n_i, n_j) \\ & Eligibility(v_i, v_j) = true \\ & Consistency(v_i, v_j) = true \\ False & otherwise \end{cases} \qquad \text{---- (1)}$$

For color RGB images, merge the regions R_i if and only if the predicate is true for each channel, independently of the others. After evaluating the merging, predicate gives $\sum_{i=i}^{n} w_i = 2$ extremity pixels the number of times it was selected for merging.

Now let us now define the distance function between regions as

$$d_{i,j} = D\big(f(R_i), f(R_j)\big) > 0 \qquad \text{---- (2)}$$

Determine the minimum distance regions

$$(i^*, j^*) = \arg\min_{i,j \in M}\{d_{i,j}\} \qquad \text{---- (3)}$$

Where i^* and j^* are the two feature vector which is extracted from the region R. Subsequently, the minimum distance of the merge regions are,

$$R_{i*} \leftarrow R_{i*} \cup R_{j*}$$

Remove the unused region

$$M \leftarrow M - \{j^*\}$$

This recursion generates a binary tree.

Clustering can be terminated when the distance exceeds a threshold, that is,

$$d_{i^*,j^*} > Threshold \Rightarrow \text{Stop Clustering}$$

In a different number of clusters, have different threshold result. On the hand diameter or radius of the cluster. For various diameter has a different threshold value.

Let $R_m \subset S$ be the region of the image, where $m \in M$. The term M is the simple invariant which is used to control the segmentation error.

The partitions of the image are represented by $\{R_m / m \in M\}$ satisfies, $\forall\ m \neq i,\ R_m \cap R_i = \phi$.

Otherwise $\bigcup_{m \in M} R_m = S$

Each region R_m has features that characterize it. Any two regions may be merged into a new region.

$$R_{new} = R_i \cup R_j \qquad \text{---- (4)}$$

Distance between region centers is defined as

$$d_{i,j} = \frac{N_i}{N_{new}} \left| C_i - C_{new} \right|^2 + \frac{N_j}{N_{new}} \left| C_l - C_{new} \right|^2 \qquad \text{---- (5)}$$

Here $N_i = |R_i|$

Where N_i denotes the number of pixels of the regions R_i

C_i denotes the number of region center of R_i

$$C_i = \frac{1}{N_i} \sum_{s \Sigma Ri} s \text{ ---- (1)}$$

$$N_{new} = N_i + N_j \qquad \text{---- (6)}$$

$$c_{new} = \frac{N_i C_i + N_j C_j}{N_{new}} \qquad \text{---- (7)}$$

The merging order is based on a measure of similarity between adjacent regions. At each step, the algorithm looks for the pair of most similar regions (the edge of minimum cost). The similarity between two regions R_1 and R_2 is defined by the following expression:

$$O(R_1, R_2) = N_1 \left\| M_{R_1} - M_{R_1 \cup R_2} \right\|_2 + N_2 \left\| M_{R_2} - M_{R1 \cup R_2} \right\|_2 \text{ ---- (8)}$$

Where

N_1 and N_2 represents the number of pixels of the regions R_1 and R_2 respectively.

$M(R)$ - The region model

$\|.\|_2$ - the L^2 norm.

$o(R_1, R_2)$-the order in which the regions have to be processed: the regions that are the more likely to belong to the same object. At each edge $e \in R_1; R_2$ is associated the value $O(R_1, R_2)$.

3.2.3 Pseudo code of proposed algorithm:

// Input: Graph $G = (V, E)$ with number of segmentations (length)

//Output: Segmented output after cluster with region merging.

For i = 1: length

Iter=1: length % cluster

For i=no.of pairs

Check the value of predicate P with respect to its neighboring regions

Sort the edge

Each time an edge e is merged, add 1 to n

If predicate P is true,

Segmentation S is constructed.

Return S

End if

End for

End for

Remark **1:** In a graph, each pair of adjacent regions is merged until the assessment of a termination criterion

Remark 2: Predicate value between two regions is true when there is exactly a cycle between them

Theorem: Let G be a graph, each pair of adjacent regions is merged until the assessment of a termination criterion. Then the segmentation S is merged.

Proof:

Let $G = (V, E)$ be an undirected graph, where $v_i, v_j \in V$ is a set of nodes corresponding to image elements. It holds the following properties:

(i) $(v_i, v_j) \rightarrow (v_i - 1, v_j)$: Region merge rate v_i

(ii) $(v_i, v_j) \rightarrow (v_i, v_j - 1)$: Region merge rate v_j

In the context of region merging, a region is represented by a component $v_{ij} \subseteq V$. The dissimilarity between two neighboring regions v_i, $v_j \subseteq V$ as the minimum weight edge connecting them. In a graph, there is a possibility to have at least one cycle. So, the region merging process will continue until the condition in Eq. (2) is not satisfied. Through this theorem grouping of pixels, pair will be evaluated, and it is stated that the condition required for grouping of image pixels are satisfied by the second equation.

Theorem:

For the probability ratio $\pi \geq 1 - O(|S|\delta)$ the segmentation S satisfying M is an over the merging of S* that is $\forall \mathrm{O} \in s^*(S), \exists R \in s(I) : \mathrm{O} \subseteq R$ s(S) set of regions of ideal segmentation, s*(S) set of regions of predicted segmentation S.

Proof:

Assume that S and S^* are any two regions in the image. Since the predicate value between two regions is true when there is exactly a cycle between them, the couple of regions (v_i, v_j)-wrongly used come from segmentation S^* and whose merging satisfies $|v_i - v_j| < w_b(v_i, v_j)$ .Here $w_b(v_i, v_j) \leq \sqrt{w_b^2(v_i) + w_b^2(v_j)}$ and the predicted merging is $p(v_i, v_j)$. Using the fact that m holds together with this property, first rebuild all true regions of S, and then eventually make some more merges: The segmentation attained over the merging of S with high probability, as claimed.

3.3 Research design and approach

This section discusses the representative methods of graph-based image segmentation. Image segmentation is the technique of allocating a label to each pixel in an image such that pixels with the same label share some image characteristics(Foreground, the background of image pixel). The result of image segmentation is a set of segments or regions that together represents the entire image. Every pixel in a region is similar with respect to certain characteristic otherwise computed property, such as color, texture, intensity, etc. Neighboring regions are meaningfully different with respect to same characteristics. This study delivers many image segmentation methods based on the color, texture, edge of the image. For each class of methods, the study provides the formulation of the problem and presents an overview of how the methods are implemented. Although, the study discusses the main differences between these methods lie in how they define the desirable quality in the segmentation and how they achieve it using distinctive graph properties.

From the literature survey, it was learned that the performance of image segmentation methods is dependent on many factors such as intensity, texture, image content. Hence, a single segmentation method cannot be applied to all types of images. At the same time, all methods do not perform well for one particular image. The graph based methods achieve segmentation on the basis of local properties of the image. To segment images for applications where detailed extraction of features is necessary, consideration of global impression along local properties is inevitable. Research in scalable, high-quality graph partitioning comparable to sequential partitioning is also lagging. Efficient implementation, energy consumption and computational time are key aspects in defining the performance of these methods.

3.4 Data Collection

In this study used a Lancet database for getting medical, scientific information in the form of images. The reason behind this dataset, it's one of the largest hosts of scientific biomedical literature is indexed in almost 5,000 scientific biomedical data. From this database, various images are collected. Particularly, have used MRI, CT scan, and X-ray images.The colored MRI images are used instead of greyscale MRI images. The gathered data were tested using segmentation approach. Through the proposed approach image occurred from the patient can be analyzed with improved level of accuracy which helps the doctor for the accurate level of disease severity.

3.5 Data Analysis

The simulation tool used here is Matlab.The semantic features of input MRI brain image are accessed by segmenting the image and representing it as a graph model.The nodes of graphs are created from blobs, not from pixels, and the edge information is denoted by the relationship between these blobs. Then normalized cuts are used to separate the image into a meaningful pattern.

3.6 Expected outcome

In this study, will discuss the useful methods of traditional, graph-based and the combination of both methods of image segmentation and the graph-theoretical approach to the image segmentation. Medical imaging is one of the most active research topics in image processing. The latest research in image segmentation has highlighted the prospective of graph-based approaches for various medical applications.The inspiration should be in the study of possessions of Minimal Spanning Tree, Euler graphs; shortest paths tree,Fuzzy graphs, Normalized cuts and Minimal cuts and revisit these ideas for image segmentation purposes. This study is helpful for those researchers who wish to carry out research in the field of image segmentation. From this, conclude that no general segmentation technique can be implemented for all types of images, but some techniques perform better than others for particular types of images indicating better performance can be obtained by selecting suitable algorithm or combination of suitable methods.

CHAPTER IV: EXPERIMENTAL METHODS

4.1 Introduction

Amongst the various segmentation approaches, the graph theoretic approaches in image segmentation make the formulation of the problem more flexible and the computation more resourceful. The problem is modeled in terms of partitioning a graph into several sub-graphs such that each of them represents a meaningful region in the image. The segmentation problem is then solved in a spatially discrete space by the well-organized tools from graph theory (Parihar & Thakur, 2014). The image segmentation problem can be interpreted as partitioning the image elements (pixels/voxels) into different categories. In order to construct a graph with an image, can solve the segmentation problem using graph theory. In this section discuss and analyze the importance of graph theory.

4.2 Application of Graph Theory to Image Segmentation

Graphs are very convenient tools for representing the relationships among objects, which are represented by vertices and relationships among vertices are represented by connections. In general, any mathematical object involving points and connections among them can be called a graph or a hypergraph. For examples, such applications include databases, physical networks, organic molecules, map colorings, signal-flow graphs, web graphs, tracing mazes as well as less tangible interactions occurring in social networks, ecosystems and in a flow of a computer program. The graph models can be further classified into different categories. For instance, two atoms in an organic molecule may have multiple connections between them; an electronic circuit may use a model in which each edge represents a direction, or a computer program may consist of loop structures. Therefore, for these examples, need multigraphs, directed graphs or graphs that allow loops. Thus, graphs can serve as mathematical models to solve an appropriate graph-theoretic problem, and then interpret the solution in terms of the original problem. At present, graph theory is a dynamic field in both theory and applications (Poghosyan, 2010).

Traditionally, a graph is modeled as a one-dimensional cell-complex, with open arcs for edges and points for vertices, the neighborhoods of a "vertex" being the sets containing the vertex itself and a union of corresponding "tails" of every "edge" (arc) incident with the vertex (Vella, 2005).

4.3 Graph Theoretical Approach to Image Segmentation

Image segmentation is a process of subdividing a digital image into its synthesized regions or objects which are useful for image analysis and has a wide variety of applications in security, forensic, medical and so on.Segmentation subdivides an image into its constitute objects. The level of subdivision depends on the type of problem solved. In this research examined about-about the image segmentation approach in various aspects in terms of flexibility and computational performance. Further, this research focused on analyzing the performance of graph theory on image segmentation approach. This paper will carry out an organized survey of many image segmentation techniques which are flexible, cost effective and computationally more efficient and finally discuss the application of graph theory which will be a highly efficient and cost-effective way to perform image segmentation.

Graph theory and discrete mathematics are related to each other since discrete mathematics is the branch of mathematics which studied about the discrete objects. These discrete objects are represented by the binary representation of objects like 0’s and 1’s. Therefore, both computer structure and operations can be described by discrete mathematics. This makes an efficient tool for improving reasoning and problem-solving skills. Concepts from discrete mathematics are useful for describing objects and problems algorithmically and analyze the time and space complexity of computer algorithms and programming languages. In the following subsection have briefly review and discuss the various segmentation methods.

4.3.1 Image segmentation

As already said, image segmentation is the process of subdividing the image or partitioning the image into its synthesized regions or objects which are useful for image analysis. Based on methods used for segmentation, it can be broadly classified into three categories; traditional segmentation, graphical theory based segmentation and a combination of both.Traditional methods approached the problem either from localization in class space using region information or from localization in position, using edge or boundary information.The segmentation method for monochrome images is mainly based on the two properties of gray level value; similarity and discontinuity.The first will partition the image based on similarity and the last based on dissimilarity.Thresholding, histogram comes under first, and split and merge come under second.Graphical methods are becoming more popular because they are providing a common framework for designing segmentation algorithms for a wide variety of applications, and also, they can be used in many prototypes. Graphical methods include Normalized cuts and image segmentation, Efficient graph-based, Iterated graph cuts for image segmentation, using minimal spanning trees and using Euler graphs.For improving the performances, both methods are used in combination. The diagram for classification of image segmentation is shown below

4.4 Image segmentation Methods

Many traditional classification methods are available for image segmentation they are discussed in the following subsections

4.4.1 Threshold Technique

In image segmentation process thresholding method is widely adopted since it is useful in the separating foreground from the background. By selecting suitable threshold value T, the gray level image can be transformed into a binary image. Thresholding creates binary images from grey-level ones by revolving all pixels below some threshold to zero and all pixels approximately that threshold to one.If g(x, y) is a threshold of f(x, y) at some global threshold T, it can be defined as,

$$g(x,y) = \begin{cases} 1, & if\ f(x,y) > T \\ 0, & if\ f(x,y) \leq T \end{cases}$$

Thresholding operation,

T = M [x, y, p(x, y), f (x, y)]

Where T- threshold;

f (x, y)- the gray value of the pixel

(x, y) , p(x, y) - property of the point (Gray value)

4.4.1.1 OTSU thresholding

Because the ease of implementation and the relative complexity, Otsu threshold is used in many applications from medical imaging to low level of computer vision, it is based on the threshold for partitioning the pixels of an image into two classes C0 and C1 (e.g., objects and background) at grey level t, where : C0 = {1, 1, 2,…, t} and C1 = {t + 1, t +2,….l - 1}, and let $q1$ and $q2$ represent the estimate of class probabilities defined as follows:

$$q_1(t) = \sum_{i=1}^{t} p(i) \qquad q_2(t) = \sum_{i=t+1}^{t} p(i)$$

And sigmas are the individual class variances defined by:

$$\mu_1(t) = \sum_{i=1}^{t} \frac{iP(i)}{q_1(t)} \qquad \mu_2(t) = \sum_{i=t+1}^{t} \frac{iP(i)}{q_2(t)}$$

Where, $p(i)$ is the pixel intensity and i are pixel value ranges from 0 to 1 (i.e. gray to the color pixel).

Finally, the individual class variances are:

$$\sigma_1^2(t) = \sum_{i=1}^{t} [i - \mu_1(t)]^2 \frac{P(i)}{q_1(t)}$$

$$\sigma_2^2(t) = \sum_{i=t+1}^{t} [i - \mu_2(t)]^2 \frac{P(i)}{q_2(t)}$$

Here, P represents the image histogram. The problem of minimizing within class variance can be expressed as a maximizing the class variance between pixels. It can be written as a difference of total variance and within class variance:

$$\sigma^2 = \sigma_w^2(t) + q_1(t)[1 - q_1(t)][\mu_1(t) - \mu_2(t)$$

Finally, this expression can safely be maximized, and the solution is the t that maximizing $\sigma_W{}^2(t)$. The final equation is achieved by the use of otsu's method for image pixels of the various pixel value of the image.

4.3.2 K-Means Segmentation

K-Means algorithm is an unsubstantiated clustering algorithm that classifies the input data points into multiple classes based on their genetic distance from each other. This algorithm assumes the data features that form a vector space and to find natural clustering. The points is clustered nearby centroids μi∀i = 1. K that are found by minimizing the objective

$$V=\sum_{i=1}^{k}\sum_{x_i \in S_i}(x_i-\mu_i)^2$$

Where there are k clusters S_i , i = 1, 2, . . . , k and μi are the centroid or mean point of all the points $x_j \in S_{i.}$

For the k-means, segmentation computes the intensity distribution (also called the histogram) of the intensities and initialize the centroids with k random intensities. Cluster the points based on the distance of their intensities from the centroid intensities. The distance between the each pixel pair is evaluated by means of varying intensity level of the pixels.

$$c^{(i)} := \arg\min \left\| x^{(i)} \mu_j \right\|^2$$

Compute the new centroid for each of the clusters.

$$\mu_i := \frac{\sum_{i=1}^{m} 1\left\{c_{(i)}=j\right\} x^{(i)}}{\sum_{i=1}^{m} 1\left\{c_{(i)}=j\right\}}$$

Where k is a parameter of the algorithm (the number of clusters to be found), i iterates over the all the intensities, j iterates over all the centroids and μ_i are the centroid intensities.

4.3.3 Split and Merging Technique

This image segmentation method is based on a quadtree barrier of an image. Therefore, it is sometimes called quadtree segmentation method. In this method, an image is represented as a tree, which is a connected graph with a number of cycles. The technique begins at the root of the tree. If it starts with non-uniform, the split & merge algorithm have two phases; the split and the merge. In the split phase, recursively split regions into four subregions in anticipation of our homogeneity criterion is met in all subregions. Conversely, if four son-squares are identical (homogeneous), then they can be merged as some connected components. This process is called as the merging process. The segmented region is the node of a tree. This process (splitting and merging) is continued recursively so that no further splits or merges are possible.

4.3.4 Normalized Cuts Method for Image Segmentation

In this method, the image is treated as a graph partitioning problem and offers a novel global method, the normalized cut, for segmenting the graph into regions/ segments. This technique not only measures the total dissimilarity between different regions and similarity within the regions. This method can be used in many applications such as segmenting immobile images and motion sequences.

Let us consider a graph G= (V, E) is partitioned into two disjoint sets, A, B, $A \cup B = V$, $A \cap B$ by simply removing edges connecting the two parts. The degree of dissimilarity between these two pieces can be computed as the total weight of the edges that have been removed. In graph theory, it is known as a cut.

$$cut\ (A, B) = \sum_{u \epsilon A, v \in B}^{n} w(u, v)$$

The normalized cut segmentation is defined as the value of total edge weight connecting the two partitions; the measure computes the cut cost as a fraction of the total edge connections to all the nodes in the graph. This disassociation measures the normalized cut and it is mathematically written as,

$$Ncut\ (A, B) = \frac{cut\ (A, B)}{assoc(A, V)} + \frac{cut(A, B)}{assoc(B, V)}$$

The normalized cut segmentation should possess two points, the first one is, a cut penalizes large segments, and another one is, fix by normalizing for the size of segments.

$$Ncut\ (A, B) = \frac{cut\ (A, B)}{Volume(A)} + \frac{cut(A, B)}{Volume(B)}$$

Where, volume (A) = sum of costs of all edges that touch A,

Volume (B) = sum of costs of all edges that touch B

$$assoc(A,V) = \sum_{u \epsilon A, t \in V} \mathrm{c(u,t)}$$

The minimization of Equation can be formulated as a generalized eigenvalue problem, which has been well-studied in the field of spectral graph theory.

NCUT Segmentation Algorithm

1. Set up problem as G = (V,E) and define affinity matrix A and degree matrix D
2. Solve (D – A)x = λDx for the eigenvectors with the smallest eigenvalues
3. Let x^2 = eigenvector with the 2 nd smallest eigenvalue λ^2
4. Threshold x^2 to obtain the binary-valued vector $x^{\prime 2}$ such that ncut($x^{\prime 2}$) ≥ ncut(x t ^{2}) for all possible thresholds t
5. For each of the two new regions, if ncut< threshold T, then recourse on the region

4.3.5 Efficient Graph-Based Image Segmentation

In the graph-based approach, a segmentation S is a partition of V into components, and its corresponding each component C ∈ S corresponds to a connected component in a graph G' = (V, E '), where E' ⊆ E. In graph based segmentation it is induced by a subset of the edges in E to measure the quality of elements in different ways with general components dissimilarities. This means that edges between two vertices in the same component should have relatively low weights, and edges between vertices in different components should have higher weights.

The pairwise predicate for segmentation is,

$$D(C_1 - C_2)\{\underset{false}{true} if \underset{otherwise}{Dif}(C_1, C_2) > \min t(C_1, C_2)$$

Where Dif(C1 , C2) - the difference between two components.

Mint (C1 , C2) is the internal different in the components C1 and C2

The region comparison predicate evaluates if there is evidence for a boundary between a pair or components by checking if the difference between the components, Dif (C_1, C_2), is large relative to the internal difference within at least one of the components, Int (C_1) and Int (C_2).

$$Dif(C_1,C_2) = \min_{v_i \in C_1, V_j \in C_2, (V_i,V_j) \in E} \in (V_i, V_j)$$

The different between two components is the minimum weight edge that connects a node vi in component C1 to node vj in C2

$$Int(C) = \max_{e\omega MST(C,E)} \omega(e)$$

Here Int(C) is to the maximum weight edge that connects two nodes in the same component.

This method Capture perceptually important Groupings and is highly efficient. The simulation results are shown in figure 27. The implementation steps are discussed as follows.

Algorithm Implementation Steps:

The input is a graph G = (V, E), with n vertices and m edges. The output image is a segmentation of V into components S = (C_1, . . . , C_r).

0. Sort E into π = (o_1, . . . , o_m), by non-decreasing edge weight.

1. Start with a segmentation S^0, where each vertex v_i is in its own component.

2. Repeat step 3 for q = 1, . . . , m.

3. Construct S_q using S_{q-1}

4. Return S = S_m.

A segmentation S is a partition of V into components such that each component (or region) C ∈ S corresponds to a connected component in a graph G' = (V, E '), where E' ⊆ E.

Goal is to have the elements in one component to be similar, and elements in different components to be dissimilar

4.3.6 The Iterated Graph Cuts Method

The iterative graph cut algorithm starts from the sub-graph that comprises the user labeled foreground/background regions and works iteratively to label the nearby un-segmented regions. In each iteration, only the local neighboring regions to the labeled regions are complicated in the optimization so that much interference from the far unknown regions can be significantly reduced.

Let use consider the smallest possible sub-graph is selected, and graph-cut is run, and the residual graph is obtained. Then, solution for a subset of connected nodes R having the same segmentation result cannot be changed simultaneously by the external flow. These changes correspond to the flipping the label of all nodes in region R.

If F is foreground,

$$\sum_{i \in R} \omega_{iS} - \omega_{iT} > \sum_{iR, j \notin R} \omega_{ij}$$

If R is background,

$$\sum_{i \in R} \omega_{iT} - \omega_{iS} > \sum_{j \notin R, i \in R} \omega_{ji}$$

w_iS and w_iT denote the terminal weight of node i.

This condition holds since the cost of the changing the solution is larger than the cost of cutting all the non-terminal edges. Solution to the part of the nodes in R might still change; however, the result of the whole R cannot change.

Algorithm

The input is mean shift initial segmentation of the given image and a graph G whose nodes consist of the user input foreground/background seed regions R.

The output is the segmentation result.

1. Add neighboring regions of foreground regions into G.
2. Construct foreground and background data models from seed regions R.
3. Use graph cuts algorithm to solve argmin f
4. Add background and foreground regions resulting from step 3 into R.
5. Add adjacent regions of the foreground seeds into G.
6. Go back to step 2 until no adjacent regions can be found.
7. Set labels of the remaining regions

4.3.7 Segmentation Using Minimal Spanning Trees

A graph G = (V, E), a spanning tree is a tree that spans all the nodes. In other words, it is a tree on all the nodes V. Evert connected graph has a spanning tree. Some graphs may have several spanning trees. In fact, you already saw Cayley's formula, which says that the complete graph K_n on n vertices has n^{n-2} spanning trees.

Formally, for a graph G = (V, E), the spanning tree is E '⊆ E such that:

- ∃u ∈ V: (u, v) ∈ E' ∨ (v, u) ∈ E' ∀v ∈V, In other words, the subset of edges spans all vertices

Need to find the spanning tree with the least cost, where the cost of the spanning tree T = (V, E') is $\sum e \in E'$, c_e, the sum of its edge costs. Frequently the minimum-cost spanning tree gets shortened to "minimum spanning tree" or "MST".

In order to addresses, the problem of segmenting Kruskal's algorithm is applied to an image into regions by defining a predicate for measuring the evidence for a boundary between two regions using a graph-based representation of the image. An important characteristic of the method is its ability to preserve detail in low-variability image region while ignoring detail in the high-variability image region. This algorithm finds a minimum spanning tree for a connected weighted graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. If the graph is not connected, then it finds a minimum spanning forest (a minimum spanning tree for each connected component).

The image to be segmented is subjected to background elimination and then represented as an undirected weighted graph G. Here each pixel is measured at one vertex of the graph and the edges are drawn based on the 8-connectivity of the pixels. The weights are assigned to the edges by using the absolute intensity difference between the adjacent pixels. The segmentation is achieved by effectively generating the Minimal Spanning Tree and adding the non-spanning tree edges of the graph with selected threshold weights to form cycles sustaining the certain criterion, and each cycle is treated as a region. The neighboring cycles recursively merge until the stopping condition reaches and obtains the optimal region based segments. This proposed method is able to locate almost proper region boundaries of clusters and is applicable to any image domain.

Algorithm for finding MST using Kruskal's method

1. Sort all the edges in an image as the non-decreasing order of weight.
2. Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If the cycle is not formed, include this edge. Else, discard it.
3. Repeat step#2 until there are (V-1) edges in the spanning tree.

4.3.8 Image Segmentation Using Euler Graphs

A closed walk in a graph G containing all the edges of G is called a Euler line in G. A graph containing a Euler line is called an Euler graph. For instance, know that a walk is always connected. Since the Euler line (which is a walk) contains all the edges of the graph, a Euler graph is connected except for any isolated vertices the graph may contain. As isolated vertices do not contribute anything to the understanding of a Euler graph, it is assumed now onwards that Euler graphs do not have any isolated vertices and are thus connected.

For example that a graph has a Euler path P with vertex v other than the starting and ending vertices, the path P enters v the same number of times that it leaves v. Therefore, there are 2s edges having v as an endpoint.

This algorithm for image segmentation problem uses the concepts of Euler graphs in graph theory. By treating the image as an undirected weighted non-planar finite graph (G), image segmentation is handled as graph partitioning problem. This method discovers region boundaries or clusters and runs in polynomial time. Subjective comparison and objective evaluation show the efficiency of the proposed approach in different image domains.

Algorithm

Step-1: Representation of image as a grid graph

Step-2: Conversion of grid graph into Eulerian

Step-3: Segmentation Procedure

Step-4: Refinement of segments

4.3.9 Segmentation Method Based On Grey Graph Cut

The image segmentation method based on gray graph cut is used for improving the performance of image segmentation, which integrates gray theory and graphs cut theory. Here the image is taken as a weighted undirected graph after that the relationships of grey-levels and positions in local regions are discussed via gray relational analysis, a gray weight matrix is established, based on which a gray partition function is constructed. Next, the image is binarized with the gray-level that corresponds to the minimum value of the gray partition function.

4.3.10 Combination of Watershed and Graph Theory

The extracting object of interest in medical images is challenging since strong noise, poor gray-scale contrast, blurred margins of tissue are characteristics of medical images. A segmentation approach that syndicates watershed algorithm with graph theory is proposed in this paper. This algorithm reconstructs gradient before watershed segmentation, based on the reconstruction, a floating-point active image is introduced as the reference image of the watershed transform.

4.3.11 the Combination of Iterated Region Merging and Localized Graph Cuts Segmentation

The Peng *et al*. (2010) Graph cuts technique provides a globally optimal solution to image segmentation; however, the complex content of an image makes it hard to precisely segment the whole image all at once. The iterated conditional mode (ICM) proposed by

Besag (1993) is a deterministic algorithm which maximizes local conditional probabilities sequentially. It uses the "greedy" strategy in the iterative local maximization to approximate the maximal joint probability of a Markov Random Field (MRF). Inspired by ICM, consider the graph cuts algorithm in a "divide and conquer" style: finding the minima on the sub-graph and extending the sub-graph successively until reaching the whole graph. The proposed method works iteratively, in place of the previous one-shot graph cuts algorithm (Boykov & Jolly, 2001).

The proposed iterated region merging method starts from the initially segmented image by the modified watershed algorithm.In each iteration, new regions which are in the neighborhood of newly labeled object and background regions are added into the sub-graph, while the other regions keep their labels unchanged. The inputs consist of the initial segmentation from watershed segmentation and user marked seeds. The object and background data models are updated based on the labeled regions from the previous iteration.

Furthermore, evaluated the segmentation performance of the proposed method in comparison with the graph cuts algorithm (Boykov & Jolly, 2001) and GrabCut (Rother *et al*., 2004). Since they used watershed for initial segmentation, for a fair comparison, also extend the standard graph cuts to a region based scheme, i.e. use the regions segmented by watershed, instead of the pixels, as the nodes in the graph. GrabCut algorithm is also an interactive segmentation technique based on graph cuts and has the advantage of reducing user's interaction under complex background. It allows the user to drag a rectangle around the desired object. Then the color models of the object and background are constructed according to this rectangle. Hence, in total, have four algorithms in the experiments: the pixel based graph cuts (denoted by GCp), the region based graph cuts (GCr), the GrabCut and the proposed iterated region merging method with localized graph cuts (Peng *et al*., 2011b).

The iterated region merging based graph cuts algorithm which is an extension of the standard graph cuts algorithm. Graph cuts address segmentation in an optimization framework and finds a globally optimal solution to a wide class of energy functions. However, the extraction of objects in a complex background often requires a lot of user interaction. The algorithm starts from the user labeled sub-graph and works iteratively to label the nearby un-segmented regions. In each iteration, only the confined neighboring regions to the labeled regions are complicated in the optimization so that much interference from the far unknown regions can be significantly reduced. Meanwhile, the data models of the object and background are updated iteratively based on high confident labeled regions. The sub-graph requires less user guidance for segmentation and thus better results can be obtained with the same amount of user interaction. Experiments on benchmark datasets validated that this method yields much better segmentation results than the standard graph cuts and the GrabCut methods in either qualitative or quantitative evaluation.

Iterated region merging with localized graph cuts Algorithm1:

1. Build object and background data models based on labeled regions Ro and Rb.
2. Build subgraph G′ =< V ′ , E′ >, where V ′ consist of Ro, Rb, and their adjacent regions.
3. Update object and background data models using the SelectLabels() algorithm
4. Use graph cuts algorithm to solve the min-cut optimization on G′ , i.e. arg min f U(f|d, f k N).
5. Update object regions Ro and background regions Rb according to the labeling results from step 4.
6. Go back to step 2, until no adjacent regions of Ro and Rb can be found.
7. **7.** Return the segmentation results.

4.3.12 The Combination of Fuzzy and Graph-Theoretical Clustering Segmentation

Clustering is a process for classifying objects or patterns in such a way that samples of the same group are more similar to one another than samples belonging to different groups. Many clustering strategies have been used, such as the hard clustering scheme and the fuzzy clustering scheme, each of which has its own special characteristics. The conventional hard clustering method restricts each point of the dataset to exclusively just one cluster. As a consequence, with this approach the segmentation results are often very crisp, i.e., each pixel of the image belongs to exactly just one class. However, in many real situations, for images, issues such as limited spatial resolution, poor contrast, overlapping intensities, noise and intensity inhomogeneities variation make this hard segmentation a difficult task. Thanks to the fuzzy set theory (Zadeh, 1965) was proposed, which produced the idea of partial membership of belonging described by a membership function; fuzzy clustering as a soft segmentation method has been widely studied and successfully applied in image segmentation (Kwon *et al*., 2003; Tolias *et al*., 1998; Tolias & Panas, 1998; Noordam *et al*., 2000; Ahmed *et al*., 2002; Zhang & Chen, 2003; Li *et al*., 2003; Pham & Prince, 1999).

Among the fuzzy clustering methods, the fuzzy c-means (FCM) algorithm (Bezdek, 1981) is the most popular method used in image segmentation because it has robust characteristics for ambiguity and can retain much more information than hard segmentation methods (Bezdek *et al*., 1993). Although the conventional FCM algorithm works well on most noise-free images.

Graph-theoretic definition of a cluster

The data to be clustered as an undirected edge-weighted graph with no self-loops G = (V,E,w), where V = {1,...,n} is the vertex set, E ⊆ V × V is the edge set, and w : E → IR∗ + is the (positive) weight function. Vertices in G correspond to data points; edges represent neighborhood relationships, and edge-weights reflect similarity between pairs of linked vertices (Pavan & Pelillo, 2003). As customary, represent the graph G with the corresponding weighted adjacency (or similarity) matrix, which is the n × n symmetric matrix A = (aij) defined as:

$$aij = \left\{ {}^{w(i,j),}_{0,} \; if\,(i,j) \in E \right\} \text{otherwise.}$$

Graph theoretic technique for metric modification such that it gives a much more global notion of similarity between data points as compared to other clustering methods such as k-means. It thus represents data in such a way that it is easier to find meaningful clusters on this new representation. It is especially useful in complex datasets where traditional clustering methods would fail to find groupings (Trivedi, 2012).

In the process of segmentation using traditional graph-theoretical clustering method is sensitive to noise and fuzzy edges. Thus false segmentation result appears. Further, the large computational complexity also affects its application.

FCM Algorithm:

Step 1: Set the cluster centroids vi according to the histogram of the image, fuzzification parameter q (1 ≤ q < ∞), the values of c and ε > 0.

Step 2: Compute the histogram.

Step 3: Compute the membership function

Step 4: Compute the cluster centroids

Step 5: Go to step 3 and repeat until convergence.

Step 6: Compute the a priori probability with the obtained results of membership function and centroids.

Step 7: Recomputed the membership function and cluster centroids with the probabilities.

Step 8: If the algorithm is convergent, go to step 9; otherwise, go to step 6.

Step 9: Image segmentation after defuzzification and then a region labeling procedure is performed.

Graph-Theoretic:

1. For the dataset having n data points, construct the similarity graph G. The similarity graph can be constructed in two ways: by connecting each data point to the other n − 1 data points or by connecting each data point to its k-nearest neighbors. A rough estimate of a good value of the number of nearest neighbors is log(n). The similarity between the points is matrix W.

2. Given the similarity graph, construct the degree matrix D.

3. Using D and W find Lsym.

4. Let K be the number of clusters to be found. Compute the first K eigenvectors of Lsym. Sort the eigenvectors according to their eigenvalues.

5. If u1, u2...uK are the top eigenvectors of Lsym, then construct a matrix U such that U = {u1, u2 . . . uK}. Normalize rows of matrix U to be of unit length.

6. Treat the rows in the normalized matrix U as points in a K-dimensional space and use k-means to cluster these.

7. If c1, c2 . . . cK are the K clusters, Then assign a point in the original dataset si to cluster cK if and only if the i th row of the normalized U is assigned to cluster cK.

4.5 Image Quality analysis

Basically, quality assessment algorithms are needed for mainly three types of applications. These are discussed as follows:

- For comparative analysis between different alternatives.
- For optimization purpose, where one maximize quality at a given cost.
- For quality monitoring in real-time applications.

Image Quality is a characteristic of an image that measures the perceived image degradation (typically, compared to an ideal or perfect image). Imaging systems may introduce some amounts of distortion or artifacts in the signal, so the quality assessment is an important problem. Image Quality assessment methods can be broadly classified into two categories. These are,

1. Objective measurement
2. Subjective measurement

4.5.1 Subjective measurement

A number of observers are selected, tested for their visual capabilities, shown a series of test scenes and asked to score the quality of the scenes. It is used to quantifying the visual image quality. However, subjective evaluation is usually too inconvenient, time-consuming and expensive (Varnan *et al.*, 2011; Kumar & Rattan, 2012).

4.5.2 Objective measurement

These are automatic algorithms for quality assessment that could analyze images and report their quality without human involvement. Such methods could eliminate the need for expensive, subjective studies. Objective image quality metrics can be classified according to the availability of an original (distortion-free) image, with which the distorted image is to be compared. As per Sakuldee and Udomhunsakul (2007), the objective quality measurements are save time more than subjective quality measurement (Eskicioglu & Fisher, 1995; Grgic *et al.*, 2004). The seven simple objective measurements are selected and used for this research study. The terms x(m,n) denotes the samples of the original image, $\hat{x}(m,n)$ denotes the samples of the compressed image. M and N are numbers of pixels in row and column directions, respectively.

The objective numerical measures of picture quality [6] that are based on computable distortion measures like mean average error, mean square error, root mean square error, laplacian mean square error, peak signal to noise ratio, structural content, maximum difference, average difference, normalized absolute error, structural similarity index are considered for study in this work on the original image and on the output image.

4.5.2.1 Mean Average Error (MAE)

The MAE is an average error of the absolute difference between the reference signal and the test image. The large value of Mean Average Error (MAE) means that image is poor quality. MAE is defined as follow:

$$MAE = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}\left|x(m,n) - \hat{x}(m,n)\right|$$

4.5.2.2 Mean Square Error (MSE)

The MSE is the measure of image quality index. The simplest of image quality measurement is Mean Square Error (MSE). The large value of MSE means that image is poor quality. MSE is defined as follow:

$$MSE = \frac{1}{MN}\sum_{m=1}^{M}\sum_{n=1}^{N}\left(x(m,n) - \hat{x}(m,n)\right)^2$$

4.5.2.3Root Mean Square Error

The Root Mean Square Error (RMSE) is the square root of the mean square error. It quantifies the average sum of distortion in each pixel of the reconstructed image.

$$RMSE = \sqrt{MSE}$$

4.5.2.4 Laplacian Mean Square Error (LMSE)

This measure is based on the importance of edges measurement. The large value of Laplacian Mean Square Error (LMSE) means that image is poor quality. LMSE is defined as follow:

$$LMSE = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[L(x(m,n)) - L(\hat{x}(m,n))\right]^2}{\sum_{m=1}^{M}\sum_{n=1}^{N}\left[L(x(m,n))\right]^2}$$

Where L(m,n) is laplacian Operator:

$$L\big(x(m,n)\big) = x(m+1,n) + x(m-1,n) + x(m,n+1) + x(m,n-1) - 4x(m,n)$$

4.5.2.5 Peak Signal to Noise Ratio (PSNR)

The PSNR is measured the quality of reconstructed image compared with the original image and a standard way to measure image fidelity. The small value of Peak Signal to Noise Ratio (PSNR) means that image is poor quality. PSNR is defined as follow:

$$PSNR = 10\log\frac{255^2}{MSE}$$

4.5.2.6 Structural Content (SC)

The SC it is used to compare two images in a number of small image patches the images have in common. The large value of Structural Content (SC) means that image is poor quality. SC is defined as follow:

$$SC = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N} x(m,n)^2}{\sum_{m=1}^{M}\sum_{n=1}^{N} \hat{x}(m,n)^2}$$

4.5.2.7 Maximum Difference (MD)

The difference between any two pixels such that the larger pixel appears after the smallest pixel. The large value of Maximum Difference (MD) means that image is poor quality. MD is defined as follow:

$$MD = Max\left(\left|x(m,n) - \hat{x}(m,n)\right|\right)$$

4.5.2.8 Average Difference (AD)

The term AD is the average of the difference between the reference signal and test image. A lower value of Average Difference (AD) gives a “cleaner” image as more noise is reduced and it is defined as,

$$AD = \left(\frac{1}{MN}\right)\sum_{i-1}^{M}\sum_{j-1}^{N}\left(f(i,j) - f^{1}(i,j)\right)$$

4.5.2.9 Normalized Absolute Error (NAE)

The large value of Normalized Absolute Error (NAE) means that image is poor quality. NAE is defined as follow:

$$NAE = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}\left|x(m,n) - \hat{x}(m,n)\right|}{\sum_{m=1}^{M}\sum_{n=1}^{N}\left|x(m,n)\right|}$$

4.5.2.10 Structural Similarity Index Metric (SSIM)

The structural similarity index is a method for measuring the similarity between two images [7]. The SSIM index is a full reference metric or can say the measuring of image quality based on an initial uncompressed or distortion-free image as a reference. It compares two images using information about luminous, contrast and structure. SSIM metric is designed to improve on traditional methods like PSNR and MSE, and this is calculated on various windows of an image. To evaluate image quality and the SSIM is given by equation Abdel-Salam Nasr *et al*. (2016)

$$SSIM = \frac{(2 \times \bar{x} \times \bar{y} + C1)(2 \times \sigma_{xy} + C2)}{(\sigma_x^2 + \sigma_y^2 + C2) \times (\bar{x})^2 + (\bar{y})^2 + C1)}$$

When C1 and C2 are constants. $\bar{x}, \bar{y}, \sigma_x^2, \sigma_y^2$ and σ_{xy} are given as:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

4.6 Summary

In this research analyzed about the application and performance of the graph theory on image segmentation process is examined. This section provides the overview of methods adopted in this research for image segmentation technique. In order to provide a clear description of the process flow, this chapter is organized in a structured manner. Previously in this chapter described about the overall summary of graph theory and followed by its application to image segmentation processing. Also, this chapter provides the experimental analysis which means theoretical approach adopted (N-cut segmentation, k-means clustering) for image segmentation process in this research is explained. Finally, parameters considered for image segmentation performance evaluation also analyzed. The next chapter provides simulation measurement of the image segmentation process is presented.

CHAPTER V: RESULTS AND DISCUSSION

5.1 Introduction

In this research adopted various image segmentation approach using graph theory approach was examined. In previous section experimental analysis related to the graph theory approach has been presented which is followed by examining the simulation results of the image segmentation is presented in this chapter. This research uses different graph theory technique for image segmentation process like thresholding technique, K-means clustering, split and merging method, cut segmentation, minimal spanning tree, Euler graph, fuzzy graph theory and so on. The corresponding simulation performance in image segmentation is investigated in this chapter. Further, this chapter provides the detailed description of existing research results with the obtained results with a clear description. ‘

5.2 Simulation Results

Image segmentation using graph theory approach has been carried with three different medical images like x-ray, Ultrasound images, and MRI images. This individual image sequence is segmented with the graph theory based different segmentation approaches. This section provides the detailed description of the image segmentation using graph theory-based approach for those three different medical images.

5.1.1MRI Images

The user manually selects the input image and displayed same as figure window with image path (Figure 1). Furthermore, it includes image enhancement process (Specifically focused image reduction and normalization). By the use of this approach, can able to improve the input image quality. After completion of this approach, continue to the image segmentation using approaches. Such as threshold, cluster, Split and merge, N-cut, minimal spanning tree, Euler graph, and iterated graph cut with region merging, Grey graph cut segmentation. The results are shown as the following the figure.

Figure 2 Image enhancement

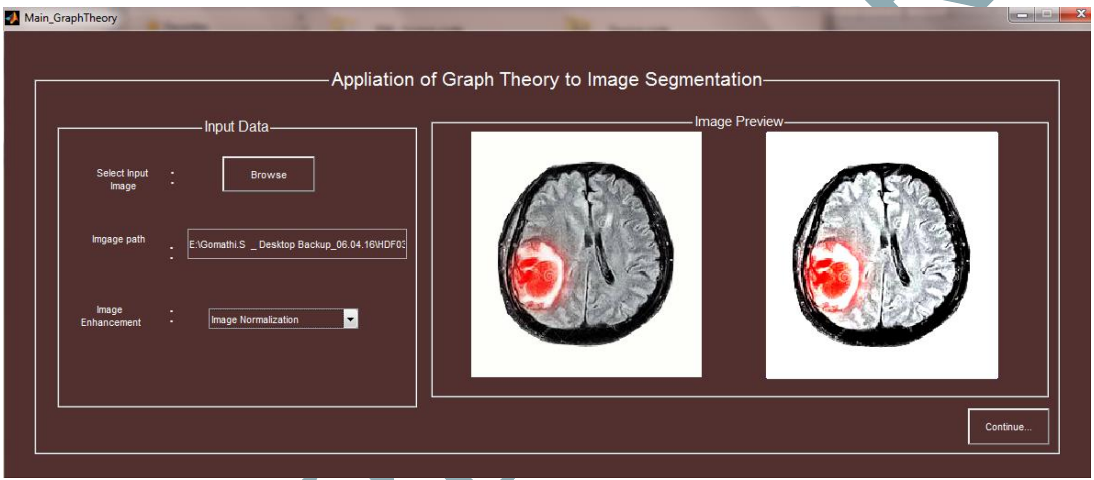
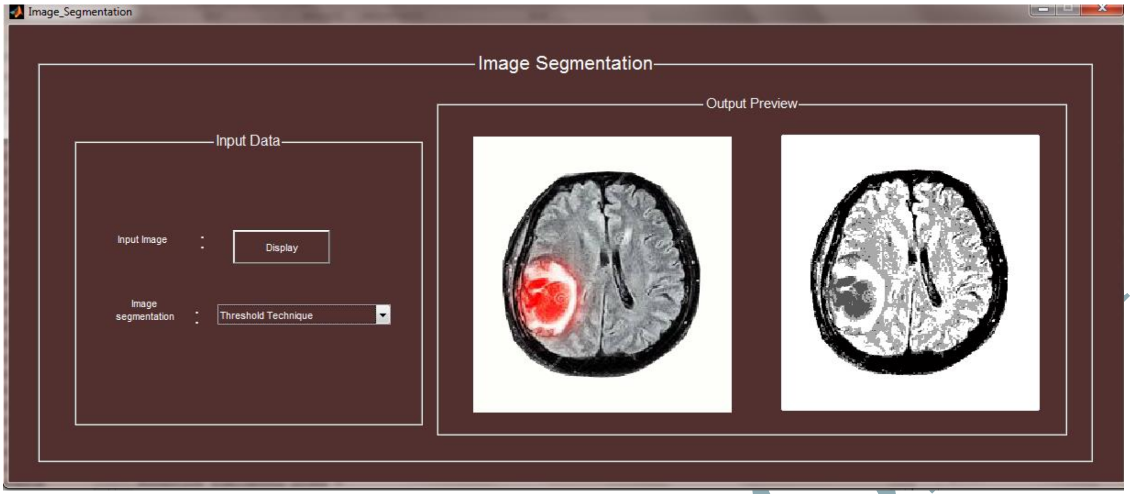
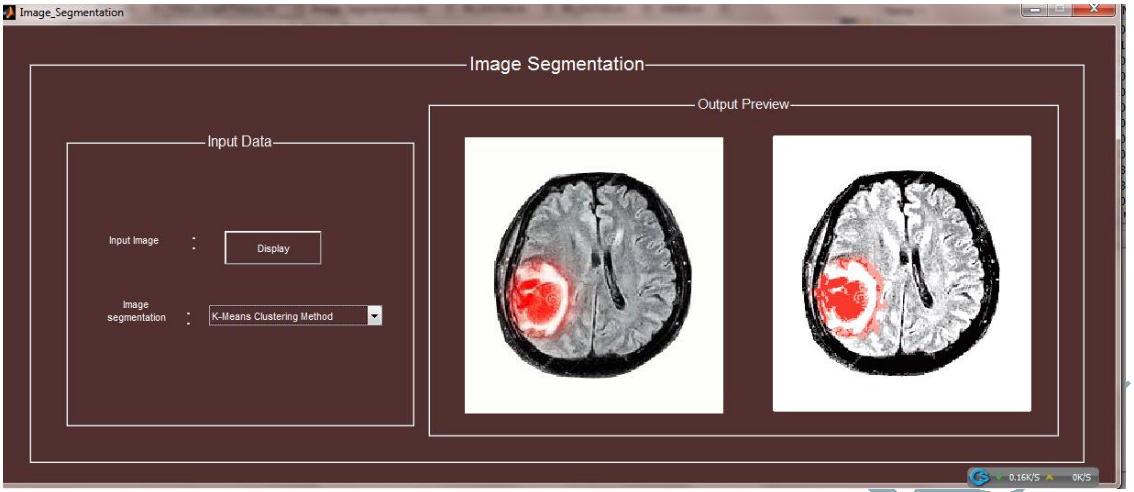


Figure 3: Thresholding:



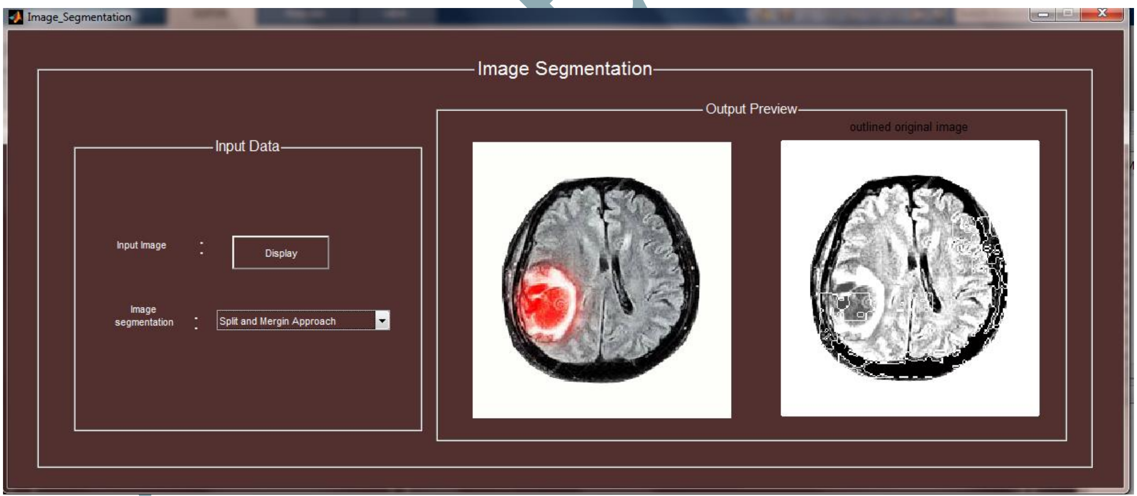
As discussed in the OSTU thresholding, need to select the single threshold value T by converting the gray level image into a binary image. Then the gray level values below T will be classified as black and above the value is white.

Figure 4: K-means Clustering



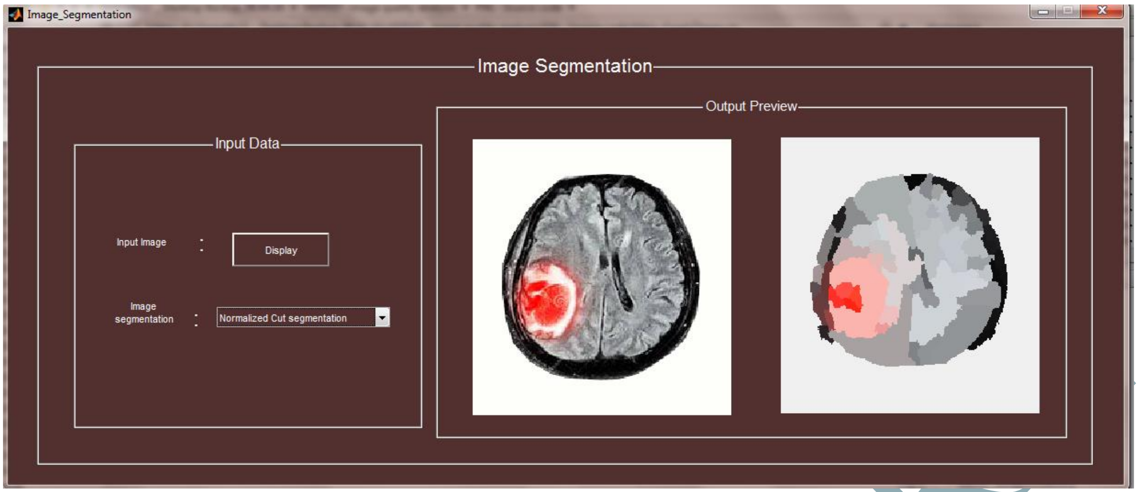
The segmented results are shown above in the figure. Here the K cluster centers are picked randomly, and need to allocate each pixel of the image to the cluster that reduces the distance between the pixel and the cluster center. Finally, calculate the cluster centers again by averaging all of the pixels in the cluster.

Figure 5 : Split and merge approach



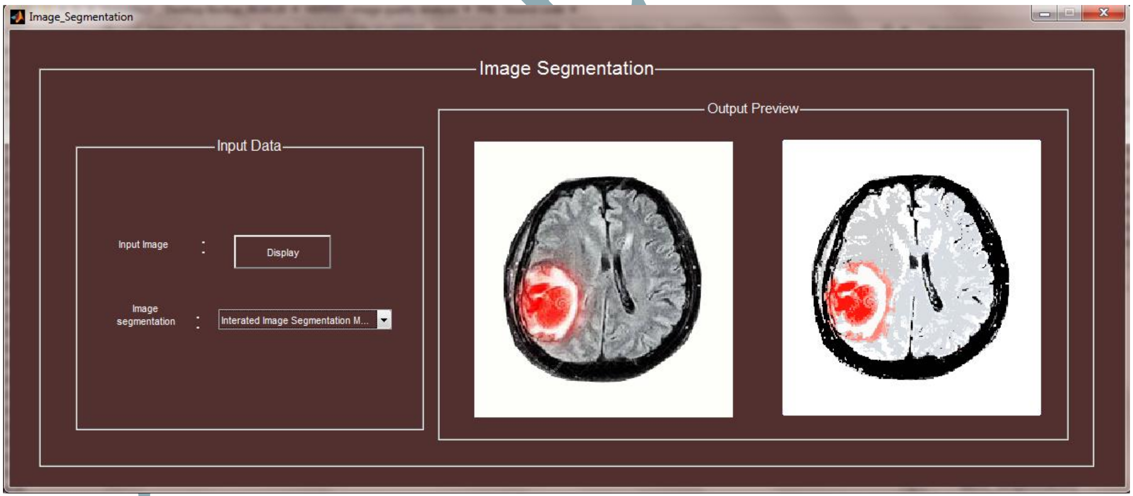
The pixels of the region are split based on their distance to the nearest height point and the plane to which that particular height point belongs. A new label is given to the pixels that belong to the new region. The new regions are stored as the new classes in the data structure, and their fields are updated. The merge process searches for neighboring regions whose associated points fall on the same plane. These are undergrown regions, and a co-planarity check determines whether they can be merged.

Figure 6: Normalized cut segmentation



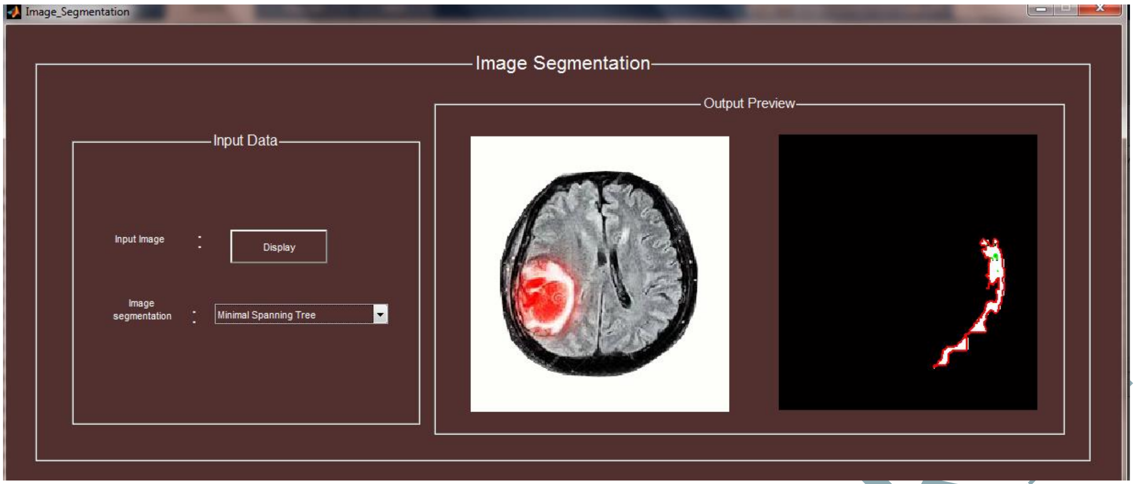
In the experimental results shows, this method successfully separates the pixel from the image, which is connected to the pixel with similar intensity. The each pixel as a node and connecting each pair of pixels by an edge. The weight on that edge should reflect the likelihood that the two pixels belong to one object.

Figure 7: Iterated Graph cut



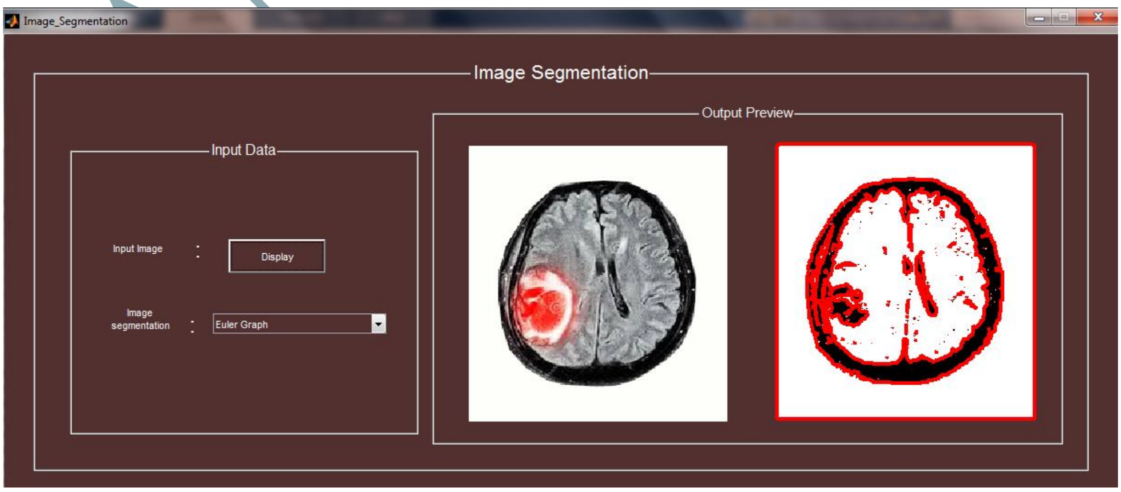
The experimental results show the black nodes represent the ground-truth foreground and grey nodes represent the ground-truth background nodes.

Figure 8: Minimal Spanning Tree



The image to be segmented is subjected to background elimination and then represented as an undirected weighted graph G. Here each pixel is measured at one vertex of the graph and the edges are drawn based on the 8-connectivity of the pixels. The weights are assigned to the edges by using the absolute intensity difference between the adjacent pixels. The segmentation is achieved by effectively generating the Minimal Spanning Tree and adding the non-spanning tree edges of the graph with selected threshold weights to form cycles sustaining the certain criterion, and each cycle is treated as a region. The neighboring cycles recursively merge until the stopping condition reaches and obtains the optimal region based segments. This proposed method is able to locate almost proper region boundaries of clusters and is applicable to any image domain.

Figure 9: Euler Graph



This algorithm for image segmentation problem uses the concepts of Euler graphs in graph theory. By treating the image as an undirected weighted non-planar finite graph (G), image segmentation is handled as graph partitioning problem. This method discovers region boundaries or clusters and runs in polynomial time. Subjective comparison and objective evaluation show the efficiency of the proposed approach in different image domains.

Figure 10: Iterated Graph cut with region merging

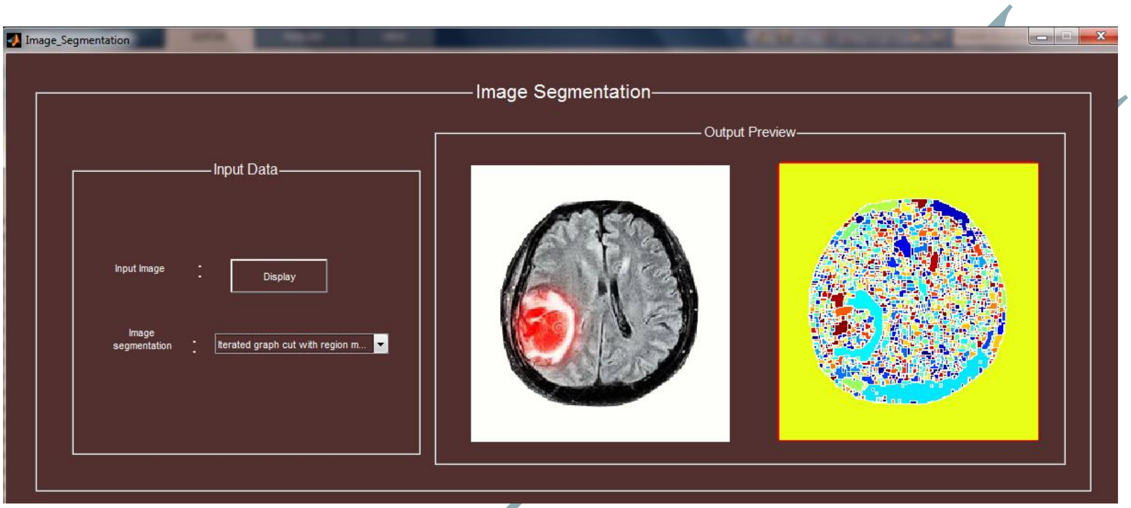
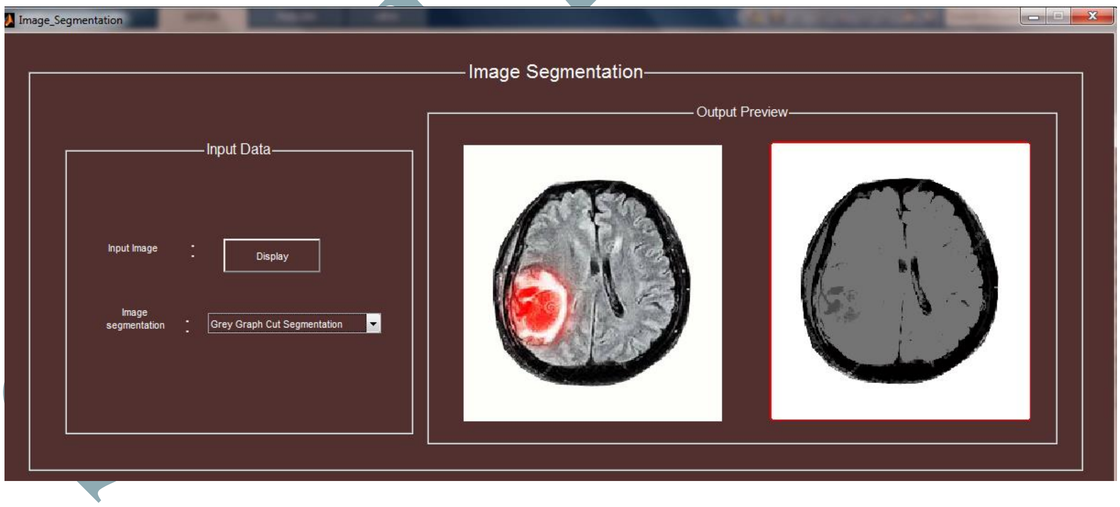
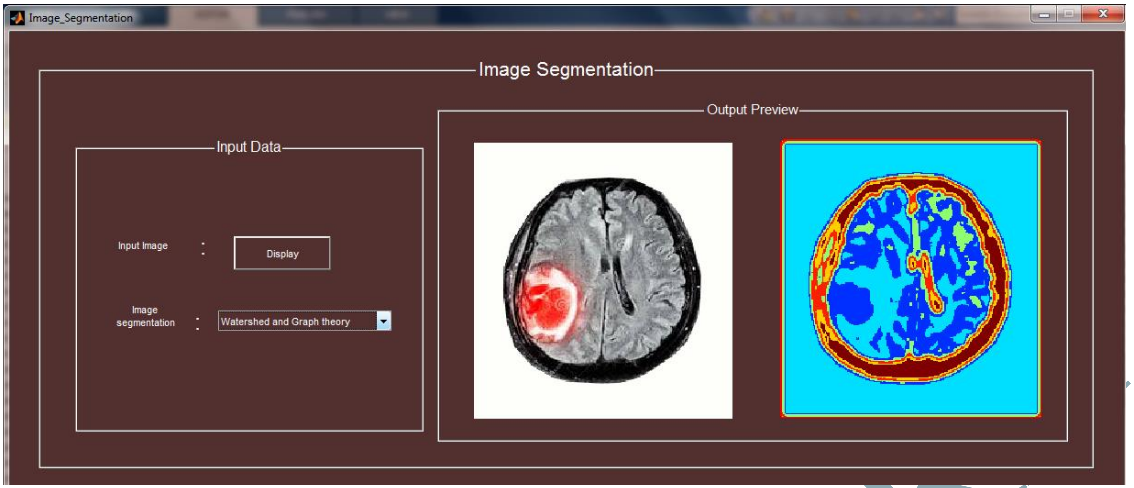


Figure 11: Grey Graph cut method



Experimental results on the visible light image and SAR image indicate this method, being superior to some existing methods like Otsu and Normalized Cut, etc., not only can segment the images with obvious difference between targets and backgrounds, but also suppress image noise effectively.

Figure 12: Watershed segmentation



The extracting object of interest in medical images is challenging since strong noise, poor gray-scale contrast, blurred margins of tissue are characteristics of medical images. A segmentation approach that syndicates watershed algorithm with graph theory is proposed in this paper. This algorithm reconstructs gradient before watershed segmentation, based on the reconstruction, a floating-point active image is introduced as the reference image of the watershed transform. Finally, a graph theory-based algorithm Grab Cut is used for fine segmentation. False contours of over-segmentation are effectively excluded and total segmentation quality significant improved as suitable for medical image segmentation.

Furthermore, verified the proposed approach using normal x-ray images of lungs and ultrasound images.The performance of these images is discussed in table 2 and table 3

Table 1: Various Quality Metrics for MRI Image

Quality Measure / Segmentation methods	MSE	PSNR	NCC	AD	SC	MD	NAE	LMSE	RMSE	SSIM
Threshold Technique	47.84	1.332	0.0159	203.2	39.1	251	0.9842	-0.75	218.74	0.0292
K-means clustering approach	49.02	1.227	0.003	205.75	65.11	254.0	0.996	-0.15	221.4	0.016
Split and merging method	9.10	18.53	1.004	-4.4416	0.9733	0	0.0215	0.1275	30.1729	0.9044
Normalized cut segmentation	39.56	12.1583	0.9194	0.1117	1.0883	238	0.1899	0.1798	62.8964	0.5125
Iterated Graph cut segmentation	49.01	1.2278	0.0040	205.7426	6.2779e+04	254.0490	0.9960	3.5050e-04	221.3860	0.0167
Minimal spanning tree	4.9403e+04	1.1933	7.6938e-06	206.5464	2.3072e+06	255	1.0000	6.0663e-04	222.2679	0.0130
Euler Graph	4.9002e+04	1.2287	0.0041	205.7256	5.8661e+04	254	0.9959	6.4021e-04	221.3632	0.0166
Iterated Graph cut with region merging	7.8040e+03	9.2077	0.8862	-0.3459	1.0748	240	0.3118	0.2438	88.3400	0.4498
Grey Graph cut	3.2053e+03	13.0722	0.8673	29.3153	1.2509	241	0.1419	0.2141	56.6150	0.6982
Watersh	2.7837e	3.68	0.353	105.8	3.7097	212	0.77	0.173	166.8	0.20

ed with graph theory	+04	46	1	839			87	0	441	98
Fuzzy with graph theory approach	2.6069e +03	13.9 695	0.811 1	39.68 05	1.4816	137	0.21 89	0.129 2	51.05 79	0.72 46
color spatial clusterin g with consensu s region merging	6.10	2.63	1.203	- 2.481 1	0.9893 3	0	0.12 15	0.103 7	26.17 29	0.98 44

Table 2: Various Quality Metrics for X-Ray Image

Quality Measure / Segmentation Methods	MSE	PSNR	NCC	AD	SC	MD	NAE	LMSE	RMSE	SSIM
Threshold Technique	2.3948e+04	4.3381	0.0168	109.9292	3.2903e+03	251	0.9838	8.6859e-04	154.7510	0.2738
K-means clustering approach	2.4579e+04	4.2251	0.0039	111.8717	6.5238e+04	254.0013	0.9961	1.5986e-04	156.7777	0.2780
Split and merging method	2.7180e+03	13.7883	1.0253	-13.1176	0.8618	0	0.1168	0.2223	52.1347	0.6870
Normalized cut segmentation	3.08+1e+10	6.832	0.9832	59.689	1.63+1e+02	57	0.068	3.56+1e-02	33.65	0.4328
Iterated Graph cut segmentation	4.7961e+02	9.7101	0.5123	22.9090	4.263	185	0.678	0.6311	21.7863	0.4106
Minimal spanning tree	2.4501e+04	4.2390	0.0057	109.9361	2.4519e+03	255	1.0000	0.0079	156.5277	0.1475
Euler Graph	2.4571e+04	4.2265	0.0041	111.8787	5.7146e+04	254	0.9961	6.5888e-04	156.7522	0.2775
Iterated Graph cut with region merging	1.6922e+04	5.8462	0.7398	-33.8956	0.8601	239	0.9463	0.3762	130.0856	0.0729
Grey Graph cut	10.7450	37.8187	0.9944	0.7599	1.0110	87	0.0068	0.0140	3.2780	0.9878
Watershe	4.6005e	11.5	0.82	-	1.2085	170	0.48	0.215	67.82	0.23

d with graph theory	+03	028	09	6.662 4			44	8	68	46
Fuzzy with graph theory approach	1.7461e +03	15.7 101	0.81 53	24.90 90	1.4263	125	0.27 28	0.131 1	41.78 63	0.61 06
color spatial clusterin g with consensu s region merging	6.7320	3.81 87	0.90 14	0.557 9	1.810	67	0.16 8	0. 1408	3.178 0	0.98 97

Table 3: Various Quality Metrics for Ultrasound Image

Quality Measure / Segmentation Methods	MSE	PSNR	NCC	AD	SC	MD	NAE	LMSE	RMSE	SSIM
Threshold Technique	7.6045e+03	9.3201	0.0206	53.7204	1.9480e+03	251	0.9845	5.1061e-04	87.2039	0.3696
K-means clustering approach	7.8661e+03	9.1732	0.0039	55.2755	6.5531e+04	254.0651	0.9961	6.9928e-05	88.6909	0.4105
Split and merging method	2.9041e+03	13.5007	1.1302	-15.4365	0.6147	0	0.2782	0.1183	53.8893	0.5989
Normalized cut segmentation	3.1404e+02	8.5694	0.5054	11.8081	12.938	161	0.4382	0.1410	28.772	0.5041
Iterated Graph cut segmentation	5.8115e+03	11.32453	12.1597	-41.3722	0.4091	130	1.9732	0.1899	51.2946	0.0327
Minimal spanning tree	7.9169e+03	9.1452	6.8306e-04	55.4562	9.0901e+04	255	0.9993	6.6296e-04	88.9770	0.4095
Euler Graph	7.8677e+03	9.1723	0.0038	55.3303	4.8693e+04	254	0.9971	4.8761e-04	88.7001	0.4097
Iterated Graph cut with region merging	1.1377e+04	7.5706	1.2690	-67.4719	0.3364	240	1.5603	0.1819	106.6620	0.0046
Grey Graph cut	1.1406e+03	17.5594	0.6854	12.8281	1.9430	171	0.2312	0.0411	33.7732	0.8041
Watersh	5.1115e	11.04	1.159	-	0.5091	170	1.07	0.119	71.49	0.05

ed with graph theory	+03	53	7	45.3722			32	9	46	87
Fuzzy with graph theory approach	2.2915e+03	1.5295	1.0972	-4.0084	0.6741	80	0.5706	0.1079	47.8700	0.4769
color spatial clustering with consensus region merging	3.1406e+02	11.5594	0.6754	13.8281	1.9690	183	0.2616	0.1421	36.7732	0.9068

5.2 Discussion

In the previous research, most of them have focused towards graph based approach to image segmentation. From these, have identified the issues and drawbacks. This is discussed as follows.Few of them have focused graph based segmentation on a medical application such as Lürig *et al*. (1997), Chen *et al*. (2006), Elmasry *et al*. (2012), Kapade *et al*. (2014), Srinivasan *et al*. (2014). However, they considered only gray scale image (only on brain or lungs image) or color image with a specific application. In specifically they not address the most challenging case of segmenting retinal layers in human eyes with multiple types of pathology from different diseases such as diabetic retinopathy, macular hole, and age-related macular degeneration. Also, they fail to determine ranges of characteristic values for interesting medical featuresLürig*et al*. (1997).On the other hand, few of them has focused towards segmentation based remote sensing application such as Mercovich *et al*. (2011), Dezső *et al*. (2012), Yang *et al*. (2015). However, this study fails to speed up the segmentation algorithm. Also, does not focus towards texture features to compute the heterogeneity. Liu *et al*. (2010), Pavan and Pelillo (2003), Elmasry *et al*.(2012),Yang *et al*. (2015)they recommended concentrating more towards pattern recognition domains such as texture segmentation, perceptual grouping, and the unsupervised organization of an image database. Also need to focus on region-based approaches. Peng *et al*. (2013) conducted a survey of graph theoretical methods for image segmentation. However, the study fails to provide versatile graph-based algorithms for a wide range of practical applications. Pham *et al*. (2014) method needed to improve especially for the extraction of representative pixels and the definition of vertex description vectors supporting the construction of graph. This proposed method does not carry the other types of VHR images. Mishra *et al*. (2014), Sarsoh *et al*. (2012) the method is sensitive to unusual cases when the background in consecutive frames changes rapidly in addition to the appearance and disappearance of multiple objects in the same scene and using multiple cameras. Kale *et al*. (2015) an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm. However, it fails to define function classes for Prim's minimum spanning tree, depth-first search, Dijkstra's shortest path algorithm, and Kruskal's minimum spanning tree. In addition need to designs for the graph traits classes will be made more generic and user defined. This way the application of all the graph classes will be truly generic, and graph theory can be applied easily for image analysis.Dikholkar *et al*. (2015)Due to that NCUT method using image pixel for segmentation, there are exponential numbers of possible partitions of the graph. As a result, it is computationally expensive to find the optimal partition (Zhao, 2015). The limitation of the Graph cut optimization approach is that it requires the number of partitions to be provided by users and hence cannot fully automatically segment an image. How to automatically determine the proper partition number for different images will be studied in the future. In addition, will explore the possibility of incorporating the proposed recursive calculation scheme into other information entropy methods (Yin *et al*., 2014).

In this thesis have proposed color spatial clustering with consensus region merging for segmenting the image and segmentation results are illustrated in Fig 2-12. Furthermore, we compared the results of our algorithm on various color images with results, which executed using its default parameter. Additional images (CT scan and ultrasound images) were also used in our comparison; however, due to space consideration results of those are not presented. In this image segmentation, a merging strategy is joined the most coherent adjacent regions together. The predicted structure represents the region partition of the image and merging more adjacent regions will produce a less representative segmentation. The most similar pair of adjacent regions corresponds to the edge with the minimum cost. This cost associated with each edge is therefore very important in order to define which regions will be merged. A merging algorithm removes some of the links and merges the corresponding nodes. Subsequently, the pair of most similar regions is merged until a termination criterion is reached. Here, the merging order is based on a measure of similarity between adjacent regions. At each step the algorithm looks for the pair of most similar regions (the edge of minimum cost). Since at each merging step the edge with the minimum cost is required, the appropriate data structure to handle the edge weights is a queue. Each edge node is inserted in the queue at the position defined by the merging order. An interesting representation of the queue is a balanced binary tree which is very efficient for managing a high number of nodes with fast access, insertion, and deletion. The region termination criterion defines when the merging ends. Usually, this criterion is based on *a priori* knowledge: the number of regions and other subjective criteria involving thresholds. Ideally, for an automatic segmentation, the termination criterion should be based on image properties which define the fact that the segmentation obtained is considered as ”good”. The segmentation evaluation provides a value which decreases the better the segmentation is. Therefore merging regions gives a better representative segmentation and the evaluation criterion decreases. The edges of the queue are processed one by one until a stabilization of the segmentation evaluation criterion. In order to assess the influence of the different representations of color, have to compare the obtained segmentations in several color spaces. To see if segmentation is close to the original image, an error metric is needed. The error between the original image and the quantized image (obtained by associating the main color of each region to each segmented pixel) was used. In this research we have used various quality matrices like MAE, MSE, RMSE, LMSE, PSNR, SC, SD, AD, NAE and SSIM. The results are discussed in table 1, 2 and 3. From this analysis, the proposed has better efficiency compared to the existing segmentation method. Experimental results indicate that MSE and PSNR are very simple, easy to implement and have low computational complexities. But these methods do not show good results. MSE and PSNR are acceptable for image similarity measure only when the images differ by simply increasing the distortion of a certain type. But they fail to capture image quality when they are used to measure across distortion types. SSIM has widely used method for measurement of image quality.

CHAPTER VI: FINDINGS AND CONCLUSION

In this present research was briefly discussed and analyzed the conventional with graphical based image segmentation method using graph theory. The implementation of various image segmentation, using graph theory was done. Some of the natural images used for the segmentation which is taken from the biomedical Dataset.The image data were captured using a data collection platform based on computer vision. Images were selected arbitrarily for our pre-segment approach. Subsequently, we implemented the segmentation algorithm in MATLAB programming language based on the max-flow/min-cut code. In the experiments, investigated the effect of performance on the following aspects: (1) energy function, (2) the comparison of the method with the conventional methods, and (4) the quantitative evaluations.

For the image segmentation experiments, used color images from the biomedical database by using Google search. It contains 79 color images for training and testing. For each image, resized the input image and generated data per image. In this experiment, we first sampled each image pixels, resulting in the reshaped image of size. Then, applied graph theory to segment images. To build descriptors of pixels, we computed a local color histogram. The indexed image is converted from the RGB input image with the minimum variance color quantization of various levels. Furthermore, we applied the graph theory to image segmentation based on brightness, color, texture, or motion information. In the monocular case, construct the graph G =V; E by taking each pixel as a node and define the edge weight wij between node i and j as the product of a feature similarity term and spatial proximity term.Through the analysis of both theoretical and practical implementation, concluded as the proposed hybrid approach which is a combination of color spatial clustering with consensus region merging for segmenting the image is an efficient method for solving the image segmentation problem. Also, possible to obtain a good classification as well as segmentation accuracy. Additionally, the proposed novel segmentation algorithm that improves upon the two primary drawbacks of the existing segmentation approach. Firstly, the regions are modeled in RGB space, secondly, a natural intuitive parameter for achieving good segmentation for a wide variety of images. All these results have been obtained without sacrificing the computational efficiency. Furthermore, we have validated this algorithm by testing on a wide variety of medical images. The performance of our algorithm is clearly superior to existing algorithm in most cases. The simulation results have shown that it can be applied for major medical analysis applications. And it can also be used for general calculations. The implementation has been made flexible in order to allow it to be applied to varying problems. Looking at the results it is easy to conclude that graph cuts are by far the best algorithm because it produces the best segmentation. However, this approach has a distinct advantage because it uses user input. Additionally, proposed method run significantly faster than the existing methods because the input to the graph cuts method includes seed pixels which considerably decrease the space of possible segmentations. Although, one of the main things to notice from the table is how fast the proposed method is in comparison to the others

6.1 Advantages

- Validated the performance of various segmentation methods using different medical images like MRI, CT scan, and ultrasound images.
- High efficiency compared to existing segmentation method

6.2 Future work

Following are some of the tasks that plan to undertake in the near future to improve the results of this thesis:

- Run the segmentation algorithm on the new dataset and evaluate its performance.
- To measure the effectiveness of proposed method based on the size of input image like 5KB, 5MB, 10MB and so on.
- To apply this approach towards some other domains like environmental protection, disaster monitoring and so on.

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