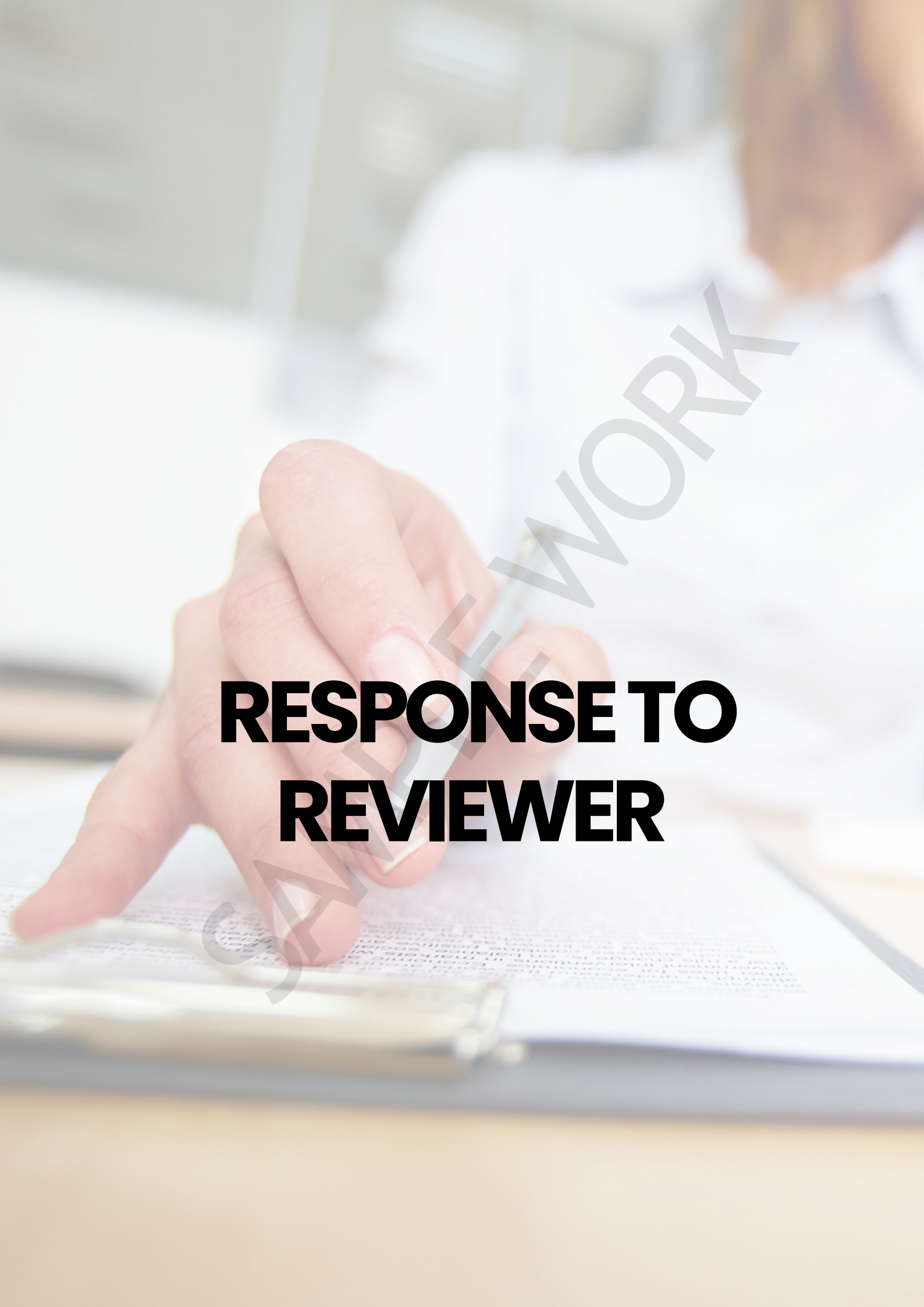

RESPONSE TO REVIEWER

SAMPLE WORK

Response to reviewer comments

Dear Editor and reviewers:

We are very appreciative of the detailed and thoughtful comments made about our submitted article. We have taken quite a while to complete the final revision. We have edited the paper now, ensuring that we address each one of the reviewer comments with care.
Moreover, we have increased the number of images for improved illustration of the concepts and added a number of new references relating to some of the most current work in the field of spectral geometry processing from 2008.
The following are detailed comments relating to all the reviewer's comments.
The reviewer comments are shown in blue, and our comments are shown in red.
Please let us know if you have any additional questions or concerns regarding the manuscript. We will happily respond to any comments that we did not address now in a timely manner.

Sincerely,

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Review #1
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Recommendation

Accept

In general, I consider that this is a very nice survey, and a timely one, since there is no contemporary work summarizing all the knowledge about spectral analysis of the meshes and their uses in graphics. I have slightly minor comments on the manuscript, and I hope that the authors will consider addressing the points I make below, even though I have recommended "accept" with no revisions. Here are my suggestions in chronological order.

It would be better to phrase the abstract and the intro in present tense (i.e. "theoretical background is provided" instead of "will be provided").

>> This is done.

Sections 1 and 2 talk about the Laplacian without ever defining it. There are indeed many definitions, as we outline later in the paper, but it would be useful to give the reader some flavor or peek ahead, before jumping into the historical survey of section 2.Otherwise, a novice reader could very well get lost -- you talk for so long on the spectrum of linear operators but define one for the first time on page 7. It might be worthwhile to move equation (1) and the related formalism to the very beginning of the exhibition.

>> This is done. We have inserted the definition of the graph Laplacian into the beginning of Section 2, and a general description of the Laplace operator at the same time. Additionally, we have inserted two new figures to provide clear illustration of the spectral approach and spectral embedding, early in the paper.

Section 2 is almost immediately followed by Section 4 (correct the numbering).

>> We are unclear on this comment since there is indeed a Section 3: Overview of the Spectral Approach.

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Page 5, top: the text gives the (false) impression that any n x n matrix M has n real eigenvalues. Add that only certain types of matrices can be diagonalized (i.e., normal matrices over the field of complex numbers or symmetric matrices over the real numbers.)

>> We could not find the text in question but suspect it may refer to the second paragraph of Section 5. This has now been addressed.

Section 2 is followed by Section 4 (fix the section numbers).

>>We are unclear on this comment because there is in fact a Section 3: Overview of the Spectral Approach.

Page 5, top: the text here gives the (false) impression that any n x n matrix M has n real eigenvalues. Add that only certain types of matrices can be diagonalized (i.e., normal matrices over the field of complex numbers or symmetric matrices over the real numbers.)

>>We could not find the text in question but suspect it might be the second paragraph of Section 5. This has now been resolved.

Page 8, very end: You state "The new operator does have some attractive properties. But a common downside...". Please elaborate on what "attractive properties" mean or remove it (it's a bit vague). The same claim occurs on page 9, top (about T": When used in an appropriate setting, T" has attractive properties" - what setting and what properties?)

>> These issues are both addressed.

Somewhere in Section 6.2 you should discuss the recent works by Wardetzky et al, namely:
"Discrete laplace operators: no free lunch". SGP 2007
"Discrete Quadratic Curvature Energies". CAGD 24, 2007.
These works have interesting and relevant discussion about the definition of the discrete Laplacian, the mass matrix and the related inner product on surfaces, symmetry, etc.

>> We put in the two references and commented on them in Section 6.2.4 and Section 6.5.

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Section 6.3.3: Q does not have a constant eigenvector because Q is constructed using the (correct) inner product on the discrete surface. This does not mean that we cannot use it for smoothing! Just that the flow needs to be constructed by taking the correct inner product into consideration.

> >> Q relates to the Tutte Laplacian T. The above operator requires the degree weighted inner product to ensure it self-adjoint and provide orthogonal eigenvectors, but Q is symmetric as it stands. The zero row sum property expresses the need for the constant vectors to lie in the kernel of the operator. If they do not, there are no choices of inner product that will fix the problem. In terms of the failure of Q to serve as a smoothing operator, we are quite specific regarding our definition of smoothing --- a low-pass filtering procedure as used in [Taubin 1995].

Page 11: When you started to talk about the cotangent Laplacian it would have nice to also mention that the cotangent Laplacian approximates the mean curvature normal (You can cite Mark Meyers thesis on this for example).

> >>We have opted to not include this interesting property of the Laplacian because it would not fit nicely into the exposition and because its relevance to spectral processing has never been discussed before.

Page 18-19, Section 9.3: It is interesting to note that with the use of a partial eigenbasis for projecting the geometry, we no-longer have the representation's rotation-invariance (for want of a better phrase) in some sense: it is not the same to first rotate the mesh (and change its global coordinates), and then project its geometry on a subset of the eigenbasis, or vice-versa (projecting it, and then rotating). At least, it seems to me that projection and rotation operators do not commute.

> >> The eigenprojection operation does commute with rotations; this fact follows directly from the associativity of matrix multiplication. If P is the n x 3 matrix of vertex positions and R is a 3 x 3 rotation matrix, then P = PRT is coordinates in rotation. Let E be the n x n matrix of orthogonal eigenvectors and let Ek be the n x k matrix of the first k columns of E. Thus, representing the eigenprojection of the vertices P using the first k eigenvectors is given by the k x 3 matrix C = EkTP. Then the rotation of these projection coefficients gives the coefficients of the eigenprojection of the rotated geometry:
>
> C = CRT =EkTPRT = EkTP.

This result has been included in Section 8.3.

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Review #3
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Recommendation

Accept Information for the Authors

Now that spectral mesh processing is possible (both in hardware and software), It is a good opportunity to publish this type of STAR. This is a significant piece of work, and a good reference (I really liked section 5, that brought together all the theorems that need to be useful). I would therefore recommend acceptance.

However, I have a little regret:

First, my philosophical two cents: The way this paper is situated itself is in completely discrete terms. The continuous setting is referred to at a number of places, but the connections between the continuous and discrete are never made explicit. I think that the derivations that Ramsay Dyer et.al provided in their tech report [DZM07] could be included in-extenso (maybe in a section that would be titled "relations between the continuous and discrete setting"). It is the first time I have ever seen that explained so clearly, so I am surprised that it is not in there, given that R. Dyer is a co-author of this paper! Since most of the geometry-processing community do not think in the "continuous setting", I could take it or leave it, but it seems to me, it would add some value to this paper.

> >> The Laplace operators are now in another section (Section 6), and the subsection on geometric Laplace operators (Section 6.5) has been restructured to emphasize the development of these operators through the properties of the differential operator, in analogy with the direction of [DZM07].

Another instance where I would invoke the continuous setting is Section 4.1 when you claim, "it is generally thought", I would state that "it is true": If one computes "circular harmonics", i.e., the eigenfunctions of the second derivative, you recover the sines and cosines of the Fourier transform exactly. Of course, this also works with a discretized circle which has a circulant graph Laplacian, yielding a discrete Fourier transform (as in Taubin's argumentation), but for me this is a "particular case" of the more general continuous setting. This is even more apparent in the case of the eigenfunctions of Laplace-Beltrami on the sphere, you recover spherical harmonics exactly.

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>> We recognize the facts highlighted by the reviewer and we added some of these to the paper. However, the original wording in the paper is intended to direct attention specifically to a similarity between the eigen analysis with respect to the Laplace-Beltrami operator and classical Fourier analysis. There is only a similarity as a major distinction exists between the two scenarios, which is noted in the second paragraph of section 4.1.

Second, I think that there could be more structure added in the exposition (again, I can do without it: since the goal of this paper is to be exhaustive, this may be anti-nomic with presenting a nice coherent structure). Historically, these are methods developed within the machine-learning community. Another way to structure all this is to start with the following reference (I think this would give some indications for answering the open questions noted at the end of the paper):

The report, written by Ham et al., aims to unify Isomap, Laplacian Eigenmaps, and LLE as special cases of kernel PCA.

It also discusses this work as continuous learning of eigenfunctions. A separate paper, titled:

Bengio et al.'s "Spectral Clustering and Kernel PCA are Learning Eigenfunctions" also unifies kernel PCA and spectral clustering as a special case of learning eigenfunctions (note the continuous context)

The taxonomy is classified as follows:

1. Learning Eigenfunctions
1.1 Kernel PCA
1.1.1 Isomap
1.1.2 Laplacian eigenmaps
1.1.3 LLE
1.2 Spectral clustering

and I believe that all the geometric methods can be placed under this taxonomy.

We can use the Kernel Trick (http://en.wikipedia.org/wiki/Kernel_trick) to describe these methods (for example in the sections about Gram matrices that come from there).

The use of these methods in geometric processing may be justified by:

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Geometric Methods for Feature Extraction and Dimensional Reduction: A Guided Tour, Christopher J.C.

Burges, November 2004, Technical Report: MSR-TR-2004-55
There have also been several recent workshops on manifold learning and geometry:

http://nips.cc/Conferences/2007/Program/event.php?ID=597

http://people.cs.uchicago.edu/~niyogi/conference/speakers.html

and I believe that Kohonen's SOMs (Self Organizing Maps) basically are a special case of those (i.e., a specialized numerical method to calculate the second and third eigenfunctions of some operator)

(Keith Van Rijsbergen's "manifold" approach to Information Retrieval probably belongs to this class of methods, too)

The reviewer provided some very good points and we're very grateful for these well-reasoned remarks. After thinking them over carefully, we chose not to make major structural changes to the current paper. The reviewer himself admits that changes are not completely warranted. Accordingly, we incorporated a couple paragraphs in Section 2.3 with reference to these relevant advancements in the machine learning community, and ostensible, included the suggested references involving kernel PCA, LLE, isomap, and Laplacian eigenmaps. Likewise, in many other places in the paper, references to works from the machine learning communities are added to reinforce ties to this important relevant study.

More detailed comments are as follows:

Section 6.6.1: Gram matrices
I would first give a general definition of a Gram matrix (i.e., matrix of dot products of a family of vectors), then specialize it to matrix learning. I would also mention the "Kernel trick" to justify the usage of these matrices.

>> Done. This is now Section 7.2.1.

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Section 2, second paragraph: I would add a citation to Belkin's paper on the convergence of graph Laplacians to their continuous types.

>> Thanks for pointing this out. However, the graph Laplacian defined in the current paper is quite particular; it is what is primarily called the Kirchhoff operator---it is combinatorial... Belkin's result applies to a graph Laplacian that is made by smoothing the pairwise Euclidean distances among points in a cloud with a Gaussian. This is an important result that we reference in Section 7.2.3, when we review non-sparse Laplacians.

Section 3: overview of the spectral approach: It is possible to obtain M by discretizing a continuous operator.

>> Added in Section 3.

"Lagrangian of a graph" -> this is very interesting (I don't know much about that), can you expand on that?

>> This is being defined in the paper now. See Section 3 after the bullet based on the operator we use.

4.1: For the "harmonic" behavior of Laplacian eigenvectors:
- please define "harmonic" (i.e. minimizer of Dirichlet energy)
- this can be explained in an intuitive way via the min-max principle (theorem 5.2 or continuous version).

>> Done.

4.3: "appropriately designed linear operator" with a specific use case in mind, how would you "design" the operator?

>> Changed to "chosen".

Section 5: It is nice to have all of these theorems in one location. Providing also a geometric intuition of what is going on would make this section even better.

>> I agree.

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End of Section 6.3.1: I think using 1/2 (T + T^t) as Levy suggested is not a very good solution. Vallet and Levy's tech report (07) gives a better solution.

>> We have now pointed out here, that the usage of a metric is a preferred way to treat T as a self-adjoint operator. We also note the work of Vallet and Levy in this context.

End of Section 6.3.2: Vallet and Levy published a version of their tech report (Euro graphics 08) with this perspective that may be of interest to the authors (it would also likely replace the citation at the end of 6.4)

>> We have developed a short discussion of the DEC in Section 6.5.1 where we reference this derivation of the Laplacian presented in [Vallet and Levy 2008]. All references to this work now refer to the Euro graphics publication rather than the technical report.

Section 6.5: it may be interesting to explain what the continuous Schroedinger operator is (and its original physical interpretation)

>> Done. Now in Section 7.1.

6.7.1: non-sparse Laplacians: Belkin's discrete Laplacian is also non-sparse and could be discussed there.

>> Precisely. This is done now, Section 7.2.3.

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Review #4
==========
Recommendation

Accept after minor revision Information for the Authors

The paper nicely summarizes recent developments in spectral mesh processing.

I had some final comments which may be of use to the authors:

A few more images of relevant applications might be helpful a plot of the rapid decay of the spectral coefficients from [KG00], the image from [Rus07] that displays the clustering of shapes with different deformations, the segmentation images from [KLT05], the nodal sets and contours of the eigenfunctions from [L06], etc.

>> While we can agree, the applications in the survey are too plentiful to choose from. Each will be beneficial from having an illustrative image, but we just cannot. After much consideration we decided to be consistent and not put up any images for the purpose of showing the results of the applications. Our main goal is for the survey to act as a single comprehensive place of reference and starting point for readers to use to delve into the related literature.

We have added new images to illustrate the principal ideas behind the spectral approach, for example: overview of the spectral approach, construction of spectral embeddings, eigenvector plots, nodal line plots, etc.

With respect to shape matching, it is important to note that there exists non-similar shapes that are isospectral.

>> We added a mention of isospectral graphs and provided a citation, in Section 10.1.

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In Section 6, it is important to point out that the geometric Laplacian can be derived as the ratio of dual edge length to primal edge length, which results in different Laplacians from the different notions of duality. This is derived and described in [Gli05].

>> We have mentioned Glickenstein's work and duality structures in a mention in Section 6.5.1. We did not flesh out the derived of the cotan operator this way, but instead pointed to the exposition in [VL08, DHLM05] and [WBH*07].

I think in Section 6, the "No Free Lunch" theorem for Laplacians should be referenced. [WMK*07]

>> This reference is added and remarked in Section 6.2.4

The MDS section I think is a bit out of place, in the operators' section.

>> It is now in a subsection, with a new title - Section 7.2.3

The generalized eigenvalues problem could also be mentioned

>> A new subsection has been added. -- Section 9.1

In section 6 it should also be mentioned that the combinatorial Laplacians are sensitive to the triangulation of the mesh.

>> A reference to the Connectivity Shapes paper has been added to highlight that the Geometric properties captured by combinatorial Laplacians are just fortunate coincidences. - second paragraph introducing Section 6,

I would have liked it if Table 1 says what each operator and its eigen structure is, each paper.

>> Perhaps. Although further discussion in the section made this clear.

There are some theoretical questions that the authors do not address, but I think are of interest:

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The mesh Laplacian can be defined for meshes that have boundaries. Does spectral analysis apply in that case?

>> In general, yes. In this survey, we have not included issues that may arise due to boundaries. Most of the items surveyed will work for closed or open meshes, if they are 2-manifolds in 3D. We have indicated this explicitly at the beginning of Section 3.

Is spectral analysis robust to small topologic perturbations- for example, will two meshes of different genus but little else different (e.g. the Homer mesh found in [Rus07]) have the same eigenvectors, eigenfunctions?

>> For operators based on pairwise geodesic distance, the eigen structures are not expected to stay stable even after small topologic perturbations. However, for mesh Laplacians we think the answer is not so clear. We mention this as an open problem for future work.

References which you should add are:

[WMK*07] Max Wardetzky, Saurabh Mathur, Felix K鋖berer, Eitan Grinspun.: Discrete Laplace operators: No free lunch. Symposium on Geometry Processing, 2007, pp. 33-37.

[Gli05] GLICKENSTEIN D.: Geometric triangulations and discrete Laplacians on manifolds. http://arxiv.org/math.MG/0508188. (2005).

>> These references have been added and discussed in the paper
-- Sections 6.2.4 and 6.5.1

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